PREFACE

SUPPLEMENT A

HYDRAULICS

This supplement is intended to augment the material of subsection 4.7 of the Hydraulics Section of the Engineering Handbook.

The objective in the preparation of Supplement A is two-fold: (a) Present improved methods for the computation of water-surface profiles which also require less effort and time, (b) Furnish working aids for the computations.

The ability to solve water-surface profiles in artificial or natural channels is required in many phases of the work of the Soil Conservation Service. Typical problems requiring such computations are: (1) The determination of tailwater depth needed in the hydraulic design of spillways in which dissipation of kinetic energy is necessary. (2) The design of artificial channels to determine their size or capacity. (3) The determination of areas of inundation for various discharges in natural drainage ways for flood damage evaluation. (4) The design of gradient control systems. (5) The design of drainage and irrigation systems.

It is expected that material improvement and saving of time in the computation of water-surface profiles can be made in the future.

Mr. Paul D. Doubt has developed major improvements in the methodology of computing water-surface profiles in the preparation of Supplement A that can effect significant savings in time spent on such work. Messrs. Richard M. Matthews and Rulon M. Jensen have helped materially with the calculations and in the preparation of the charts and examples. Miss Joan Donovan typed the manuscript. This work was done under the general direction of Mr. M. M. Culp, Head, Design Section.

July 6, 1954

SUPPLEMENT A TO SUBSECTION 4 OPEN CHANNEL FLOW

WATER SURFACE PROFILES

PART 1

INTRODUCTION AND DERIVATIONS

A number of writers have presented various methods for solving the surface profile of steady flow in open channels. Some methods are derived by integrating the differential equation of varied flow for special limiting cases or by means of certain approximations. These methods restrict their solutions to prismatic channels. Other methods make use of the slope which the differential equation defines. The slope, $dd/d\ell$, for a reach is evaluated either as an average of the slopes at the end sections or as the slope existing at some intermediate section in the reach. It is proper to use any such process of averaging for the arithmetical error may be kept as arbitrarily small as desired by choosing a sufficiently small increment of the reach.

This supplement presents methods and working aids by which water-surface profiles for steady flow in open channels may be determined as a direct solution for either prismatic or natural channels. Two methods of water-surface-profile determinations are given along with examples. The first method is associated with prismatic channels. The varied or nonuniform-flow differential equation has been rewritten in terms of two channel parameters and the discharge Q for which the profile is being determined in prismatic channels. Because of this, a number of water-surface profiles for various discharges Q may be solved for a given channel with a minimum amount of additional work. The two parameters designated as $Q_{c,d}$ and $Q_{n,d}$ are the critical and normal discharges, respectively, for the depth of flow in the channel. These two parameters are also used to determine flow condition or profile type. Thus, use is made of the parameters in two distinct ways. Charts are included to enable quick evaluation of these parameters. The second method is associated with natural channels.

Equation of varied flow. The customary basic assumptions generally postulated in the theory of varied flow are reiterated briefly in this derivation. The restrictive character of some assumptions can be relieved by approximations or corrections.

The assumptions are:

1. Steady flow conditions exist; that is, the discharge at each section remains constant and equal for the time interval under consideration.

2. Manning's formula defines the slope of the energy gradient.

3. Flat slope channels are assumed; that is, channel slopes are sufficiently flat so the following approximations are valid or acceptable.

   a. The depth of flow, which is generally measured normal to the bottom slope, may be measured vertically.

   b. Hydrostatic pressure distribution exists throughout the depth of flow. This statement requires that little vertical curvilinear movement exists.

c. The slope of the water surface is sufficiently flat so that the depth of flow measured in a vertical direction is a close approximation of the hydrostatic head.

d. No air entrainment occurs.

4. The velocity distribution in each section is nearly equal to the average velocity v such that the quantity $v^2/2g$ may be a sufficiently close expression of the kinetic energy head.

a. This requires that the cross profile of the water surface be nearly level.

5. No significant unaccountable losses occur, such as those due to bends, hydraulic jumps, and abrupt transitions.

Although the derivation of the varied flow equation may be found in many references, it will be repeated here because of the numerous forms in which it appears

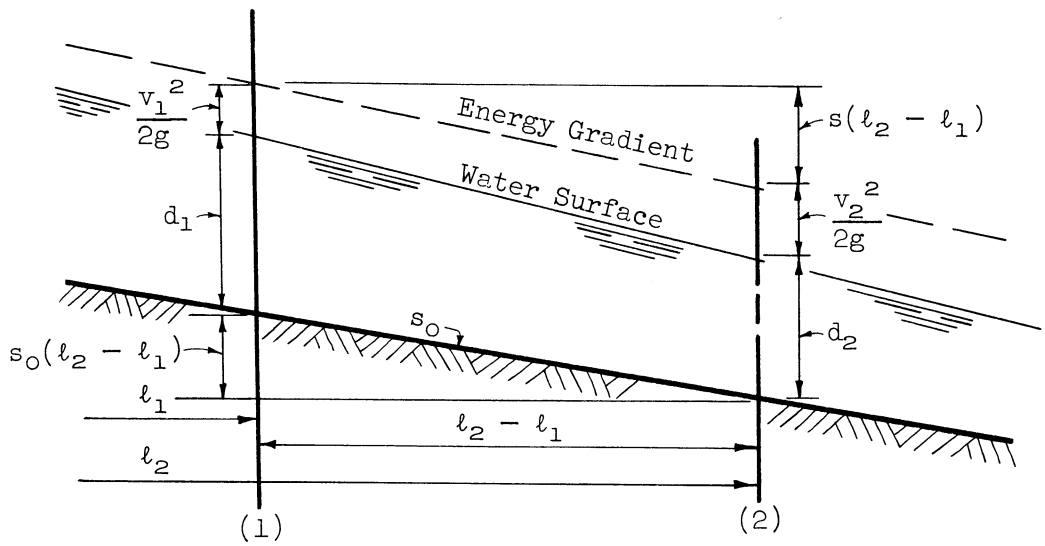


FIGURE A.1

$$d_1 + \frac{v_1^2}{2g} + s_o\,(\ell_2 - \ell_1) = d_2 + \frac{v_2^2}{2g} + s\,(\ell_2 - \ell_1)$$

Rearranging

$$(d_2 - d_1) + \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = (s_o - s)(\ell_2 - \ell_1) \qquad \text{A.1}$$

Now as $(\ell_2 - \ell_1) \equiv \Delta\ell \rightarrow 0$; then $(d_2 - d_1) \equiv \Delta d \rightarrow 0$ and

$$\left[\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right] \equiv \Delta\,\frac{v^2}{2g} \rightarrow 0$$

Hence, writing in differential form after dividing Eq. A.1 by $\Delta\ell$

$$\frac{dd}{d\ell} + \frac{d}{d\ell}\left[\frac{v^2}{2g}\right] = s_o - s \qquad \text{A.2}$$

This is a common form in which the varied flow equation is presented. The rate of change of depth of flow with respect to length of reach, $dd/d\ell$, can be written as a function of s_o, $Q_{c,d}$, $Q_{n,d}$, and Q.

Differential equation of varied flow for positive slopes. Proceeding to write the terms of Eq. A.2 in terms of the five quantities mentioned, the second term will be considered first. The term, $\frac{d}{d\ell}\left[\frac{v^2}{2g}\right]$, which expresses the rate of change in velocity head can be written by observing

$$v = \frac{Q}{a}$$

$$\frac{d}{d\ell}\left[\frac{v^2}{2g}\right] = \frac{Q^2}{2g}\frac{d}{d\ell}\left[\frac{1}{a^2}\right]$$

But

$$\frac{d}{d\ell}\left[\frac{1}{a^2}\right] = -2\frac{1}{a^3}\frac{da}{d\ell} = -2\frac{T}{a^3}\frac{dd}{d\ell}$$

for $da = T\ dd$

Therefore

$$\frac{d}{d\ell}\left[\frac{v^2}{2g}\right] = -\frac{Q^2}{g}\frac{T}{a^3}\frac{dd}{d\ell}$$

Let

$$Q_{c,d}^2 = g\frac{a^3}{T} \qquad \text{A.3}$$

Then

$$\frac{d}{d\ell}\left[\frac{v^2}{2g}\right] = -\left[\frac{Q}{Q_{c,d}}\right]^2\frac{dd}{d\ell} \qquad \text{A.4}$$

The slope of the energy gradient, s, (or the rate of energy loss due to friction per unit length of channel) is also to be rewritten. By Manning's formula

$$s = \left[\frac{nQ}{1.486\ ar^{2/3}}\right]^2 \qquad \text{A.5}$$

Further, at the same depth of flow used in evaluating Eq. A.5 there corresponds a normal discharge $Q_{n,d}$ for a channel having a positive slope s_o and a roughness coefficient n. This discharge is written

$$Q_{n,d} = \frac{1.486\ ar^{2/3}}{n}\ s_o^{1/2} \qquad \text{A.6}$$

eliminating the factor $\frac{1.486}{n}\ ar^{2/3}$ in Eqs. A.5 and A.6 obtain

$$s = s_o\left[\frac{Q}{Q_{n,d}}\right]^2 \qquad \text{A.7}$$

Substituting the terms expressed by Eqs. A.4 and A.7 into the varied flow Eq. A.2 and rearranging

$$s_o \frac{d\ell}{dd} = \frac{\left[\frac{Q}{Q_{c,d}}\right]^2 - 1}{\left[\frac{Q}{Q_{n,d}}\right]^2 - 1} \qquad \text{A.8}$$

is the differential equation of varied flow for positive-slope channels which will be considered further.

Differential equation of varied flow for channels having horizontal bottoms. The differential equations for channels having horizontal or negative (adverse) bottom slopes can be written in terms of Q, $Q_{c,d}$, and $Q_{n,d}$ if the quantity $Q_{n,d}$ is defined. The normal discharge, as defined for positive-slope channels, does not exist for channels having horizontal bottoms. This can be readily ascertained for s_o is zero in Eq. A.6 and $Q_{n,d}$ is zero. To eliminate this seemingly unfortunate situation, the quantity $Q_{n,d}$ for channels having horizontal bottoms is arbitrarily defined to be the same as though the channel has a positive slope of unity. The slope of the energy gradient is still given by Eq. A.5, but at the same depth of flow used in evaluating Eq. A.5 there corresponds a normal discharge, $Q_{n,d}$, for the same channel having a positive slope of unity and of roughness coefficient n. This discharge is written

$$Q_{n,d} = \frac{1.486\ ar^{2/3}}{n} \qquad \text{A.9}$$

Eliminating the factor $\frac{1.486\ ar^{2/3}}{n}$ in Eqs. A.5 and A.9 obtain

$$s = \left[\frac{Q}{Q_{n,d}}\right]^2 \qquad \text{A.10}$$

Then by Eq. A.2 in which s_o is zero and Eqs. A.4 and A.10, the varied flow equation for channels with horizontal bottoms becomes

$$\frac{d\ell}{dd} = \frac{\left[\frac{Q}{Q_{c,d}}\right]^2 - 1}{\left[\frac{Q}{Q_{n,d}}\right]^2} \qquad \text{A.11}$$

Differential equation of varied flow for channels having negative slopes. For channels having negative slopes, the normal discharge $Q_{n,d}$ becomes imaginary according to an evaluation of Manning's formula and the definition of normal discharge for positive slopes.

$$Q_{n,d} = \frac{1.486}{n}\ ar^{2/3}\ s_o^{1/2} \qquad \text{A.6}$$

This is readily ascertained for s_o is negative in Eq. A.6. To eliminate any element of confusion which might arise in the use of imaginary numbers, the normal discharge $Q_{n,d}$ for negative slopes will be given a definition different from that used for channels with positive slopes. This change in definition will necessarily cause a change in the varied flow differential

equation from that given by Eq. A.8. On taking the definition of the normal discharge of the same channel with a positive slope $|s_o|$ the evaluation of the slope of the energy gradient, on using the same argument as before, becomes

$$s = - s_o \left[\frac{Q}{Q_{n,d}}\right]^2 \qquad \text{A.12}$$

The varied flow differential equation for channels with negative slope is, by substituting Eqs. A.4 and A.12 into Eq. A.2,

$$|s_o| \frac{d\ell}{dd} = \frac{\left[\frac{Q}{Q_{c,d}}\right]^2 - 1}{\left[\frac{Q}{Q_{n,d}}\right]^2 + 1} \qquad \text{A.13}$$

The symbol $|s_o|$ is the absolute value (or the positive value) of the negative slope $|s_o|$ and $Q_{n,d}$ is evaluated as a real number for the channel having the positive slope $|s_o|$ at the depth of flow d.

PART 2

FLOW CONDITIONS

During the process of the solution of Eq. A.8, it will be necessary to know or be able to recognize flow conditions. Therefore, before further considering this equation, the criteria which will define flow conditions will be considered. Flow conditions have three aspects: (1) flow is subcritical, critical, or supercritical; (2) flow is subnormal, normal, or supernormal; (3) flow is retarded, constant velocity (i.e., uniform), or accelerated. As will be seen, these three aspects are not independent of one another for, in general, any two will establish the third. Flow conditions are evaluated at a particular section having a depth of flow d and a given discharge Q. The flow conditions will become apparent during the solution for a surface profile upon observing the values of the terms $Q/Q_{c,d}$; $Q/Q_{n,d}$; and $dd/d\ell$. The criteria which will define flow conditions will be discussed in the following three paragraphs. The relationship between the three aspects will be considered in the fourth paragraph. A summary of the discussion is given in table A.1, page A.8.

The critical discharge at depth d may be determined for a given channel section by the relation Eq. A.3. The value of the ratio $Q/Q_{c,d}$ can be used as a criterion to determine whether a discharge of Q flowing at a depth d is subcritical, critical, or supercritical flow. By definition, the flow of a discharge Q at the depth d is subcritical, critical, or supercritical according to whether Q is less than, equal to, or greater than the critical discharge $Q_{c,d}$ corresponding to the depth d. Hence, the general criterion may be written: the flow of a discharge Q at a depth d will be subcritical, critical, or supercritical according to whether $Q/Q_{c,d}$ is less than, equal to, or greater than unity. It is important to remember that the discharge Q at a flow depth d is compared to the datum $Q_{c,d}$ in determining the prefix to the word critical. Frequently, the depth of flow d for a discharge Q is compared to the datum d_c corresponding to the discharge Q to determine this flow

condition. An interchange of prefixes is obtained by this comparison, for when d is less than d_c the flow is supercritical and when d is greater than d_c flow is subcritical. The criterion can also be worded: the depth of flow d will be less than, equal to, or greater than critical depth of flow for Q according to whether $Q/Q_{c,d}$ is greater than, equal to, or less than unity.

The normal discharge at depth d for a given channel section of infinitesimal length, having a bottom slope s_o and roughness coefficient n, may be obtained by Eq. A.6. The value of the ratio $Q/Q_{n,d}$ can be used as a criterion to determine whether a discharge Q flowing at depth d is subnormal, normal, or supernormal flow. The flow of a discharge Q at a depth d is subnormal, normal, or supernormal according to whether Q is less than, equal to, or greater than the normal discharge $Q_{n,d}$ corresponding to the depth d. The criterion may be written: The flow of a discharge Q at a depth d will be subnormal, normal, or supernormal according to whether $Q/Q_{n,d}$ is less than, equal to, or greater than unity. Here again, it is important to remember the discharge Q flowing at the depth d is compared to the datum $Q_{n,d}$ in determining the prefix to the word normal. It is desirable to observe that the depth of flow d for a discharge Q may be compared to the datum d_n, the normal depth of flow corresponding to the discharge Q, to determine this flow condition. Once more, an interchange of prefixes is obtained by this comparison, for the flow is supernormal when d is less than d_n and when d is greater than d_n the flow is subnormal. Rewriting the criterion: the depth of flow d for a discharge Q is less than, equal to, or greater than the normal depth d_n corresponding to Q according to whether the ratio $Q/Q_{n,d}$ is greater than equal to, or less than unity.

According to the sign convention used in deriving Eq. A.8, the value $dd/d\ell$ (the instantaneous rate of change of depth with respect to the length of channel at a section) is positive when the depth of flow is increasing in a downstream direction. This imposes that flow be retarded. The criterion becomes: a discharge Q flowing at a depth d is retarded, uniform, or accelerated flow according to whether $dd/d\ell$ is positive, zero, or negative.

Equation A.8 shows that $dd/d\ell$ is positive if the ratios $Q/Q_{c,d}$ and $Q/Q_{n,d}$ are:

(a) both greater than unity, or
(b) both less than unity.

In other words, as has been shown by the three preceding paragraphs, flow is retarded for the following two sets of flow conditions:

(a') flow is supercritical and supernormal
(b') flow is subcritical and subnormal

Again Eq. A.8 reveals that $dd/d\ell$ is negative if

(c) the ratio $Q/Q_{c,d}$ is greater than unity and $Q/Q_{n,d}$ is less than unity, or
(d) the ratio $Q/Q_{c,d}$ is less than unity and $Q/Q_{n,d}$ is greater than unity.

Therefore, flow is accelerated when

(c') flow is supercritical and subnormal, or
(d') flow is subcritical and supernormal.

Certain special cases of flow conditions occur when $Q/Q_{n,d} = 1$. If $Q/Q_{n,d} = 1$ and $Q/Q_{c,d} \neq 1$ then $dd/d\ell = 0$ and flow is at normal depth or

uniform flow occurs. The uniform flow is

(1) subcritical if $Q/Q_{c,d} < 1$

(2) supercritical if $Q/Q_{c,d} > 1$.

Another set of special cases is obtained when $Q/Q_{c,d} = 1$ and $Q/Q_{n,d} \neq 1$. If $Q/Q_{c,d} = 1$ and $Q/Q_{n,d} > 1$ then $dd/d\ell = +\infty$ and

(3) flow is at critical depth and is being retarded through a hydraulic jump, i.e., flow is passing from a supercritical condition to a subcritical condition and is supernormal.

If $Q/Q_{c,d} = 1$ and $Q/Q_{n,d} < 1$ then $dd/d\ell = -\infty$ and

(4) flow is at critical depth and is being accelerated through a control section, i.e., the bottom slope of the channel increases from a slope less than critical to a slope equal to or greater than critical.

When $Q/Q_{c,d} = 1$ and $Q/Q_{n,d} = 1$

(5) the very special unstable case of uniform critical flow is obtained. In this case, $dd/d\ell$ is of the indeterminate form, 0/0.

It can be observed that any two aspects of flow conditions will not necessarily determine the third, nor are the three aspects independent for the special cases of flow condition.

A knowledge of flow conditions is required before computations can be performed properly and reliably. For instance, computations for profiles of subcritical flows are made in an upstream direction and computations for supercritical flows are made in a downstream direction. There are definite physical reasons for this statement. It has been shown that when flow is supercritical the depth of flow at a given section is unaffected by any channel elements, or change of channel elements, downstream from the section so long as the flow remains supercritical at the section under consideration. Therefore, the depth of flow at the given section is dependent only on the flow characteristics upstream from the section at which supercritical flow exists. Hence, to consider the effect of these upstream flow characteristics in deducing the value of depth of flow at a section, profile computations for supercritical flows must be carried in a downstream direction to this section.

Similarly, the depth of flow at a given section which is subcritical is not influenced by any of the channel elements or change of channel elements upstream from the section as long as the flow remains subcritical at the section. The depth of flow at the section is dependent only on the flow characteristics of the channel downstream from the section. The effects of these flow characteristics on the subcritical depth at the section can only be used by considering them in solving for the depth at the section. Thus, surface profile computations must be carried in an upstream direction in regions of subcritical flow.

When computations are made in the incorrect directions, solutions for the depth of flow diverge from the correct depth. Conversely, when computations are made in the correct direction the calculated depth of flow tends to converge to the correct depth even though the initial depth is incorrect. This fact is used in determining a depth of flow. See ES-83, Ex. 4, page A.19.

TABLE A.1

Equation A.8

Summary of Criteria and Relationship between flow conditions; positive slopes

When $\frac{Q}{Q_{c,d}}$ is → / When $\frac{Q}{Q_{n,d}}$ is ↓	greater than unity then flow is supercritical $d < d_c$ and ↓	equal to unity then flow is critical $d = d_c$ and ↓	less than unity then flow is subcritical $d > d_c$ and ↓
greater than unity then flow is supernormal $d < d_n$ and →	(a) $\frac{dd}{d\ell} > 0$ flow is retarded $d_1 < d_2$	(3) $\frac{dd}{d\ell} = +\infty$ flow is retarded (hydraulic jump)	(d) $\frac{dd}{d\ell} < 0$ flow is accelerated $d_1 > d_2$
equal to unity then flow is normal $d = d_n$ and →	(2) $\frac{dd}{d\ell} = 0$ flow is uniform $d_1 = d_2$	(5) $\frac{dd}{d\ell} = \frac{0}{0}$ = indeterminate very special unstable case of uniform critical flow. $d_1 = d_n = d_c = d_2$	(1) $\frac{dd}{d\ell} = 0$ flow is uniform $d_1 = d_2$
less than unity then flow is subnormal $d > d_n$ and →	(c) $\frac{dd}{d\ell} < 0$ flow is accelerated $d_1 > d_2$	(4) $\frac{dd}{d\ell} = -\infty$ flow is accelerated (control section)	(b) $\frac{dd}{d\ell} > 0$ flow is retarded $d_1 < d_2$

Because of these facts, the value of the ratio $Q/Q_{c,d}$ will be used to determine the direction in which computations are to be carried. Computations are to be carried in a downstream or upstream direction according to whether $Q/Q_{c,d}$ is greater or less than unity. If $Q/Q_{c,d}$ is unity, either a control section exists at some break in grade or a hydraulic jump exists or flow is uniform and critical.

PART 3

DETERMINATION OF WATER SURFACE PROFILE

The objective in the majority of water-surface-profile problems is the determination of the surface profile for a given discharge Q. Two methods are presented. The first and simpler method presented is applicable only to prismatic channels and is illustrated by Exs. 1, 2, and 3, ES-83, pages A.16 to A.18 inclusive. The second method, illustrated by Exs. 4, 5, and 6, is applicable to either prismatic or natural channels and is particularly useful in the determination of a number of surface profiles for various discharges in a given channel. Both methods give direct results, thus eliminating all trial and error procedures.

The type of problem solvable by the simpler method. The simpler method determines the length of reach between two depths of flow for a given discharge Q in a given prismatic channel. The water-surface profile solvable by this method has the following data given:

1. The discharge Q, for which the water-surface profile is to be determined.

2. The size and shape of the prismatic channel.

3. The slopes of the channel bottom and locations at which changes in grades occur.

4. Manning's roughness coefficient n.

5. Depth of flow at the downstream section of the channel, if flow is subcritical, or at the upstream section of the channel, if flow is supercritical. If the depth of flow is not given, it must be determinable. One or both of two procedures can be used to determine the depth when the depth of flow is not given. One procedure is to determine a control section. This is illustrated by Ex. 1, ES-83, page A.16. The second procedure is illustrated by Ex. 4, ES-83, pages A.19 through A.21.

The first step in solving a water-surface profile is to determine which of those breaks in grade of the channel bottom are control sections for the discharge Q, the surface profile of which is being determined. The depth of flow at a control section is the critical depth d_c, corresponding to the discharge Q. Any method of water-surface-profile determination requires that one depth of flow be given or be determinable. See sheet 4, ES-38, for definition of control sections.

The second step for the solution of a water-surface profile in prismatic channels is generally the determination of the normal depth of flow for each bottom slope of the channel for the discharge Q.

The third step is the determination of the water-surface profile itself. The direction in which the computations are to be made (see pages A.7 and A.9) is known. Equation A.8 may be rewritten in the form

$$\frac{d\ell}{dd} = \frac{1}{s_o}\,\frac{\left[\dfrac{Q}{Q_{c,d}}\right]^2 - 1}{\left[\dfrac{Q}{Q_{n,d}}\right]^2 - 1} \equiv \frac{1}{s_o}\,\frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3}s_o^{1/2}}}{\frac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}}\right]^2 - 1} = \frac{2R}{s_o} \qquad \text{A.14}$$

It is good practice to tabulate data in a manner similar to that shown by the table in ES-83, page A.16. Various values of depths are arbitrarily selected between the known depth and the normal depth of flow for the discharge Q. Observe that the identities

$$\left[\frac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right] \equiv \frac{Q}{Q_{c,d}} \quad \text{and} \quad \left[\frac{\frac{nQ}{b^{8/3}s_o^{1/2}}}{\frac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}}\right] \equiv \frac{Q}{Q_{n,d}}$$

are true. The numerator of the left hand member of each identity is found in performing the second step and the denominators can be read from ES-24 and ES-55, pages A.26 to A.29 for the selected depths d of flow. The value of R may be determined by ES-53, page A.30, or calculated by substituting into Eq. A.8.

For various values of d, a plot of corresponding values of $2R/s_o$ can be made. Such a plot is given by Fig. A.2, page A.11. In general, when $d = d_c$ the value of $2R/s_o$ is zero and for values of $d \rightarrow d_n$ the value of $2R/s_o \longrightarrow \pm\infty$, where d_n is the normal depth corresponding to the discharge Q. The value of $2R/s_o$ is positive when flow is retarded and negative when flow is accelerated. The value of $2R/s_o$ is also the value of $d\ell/dd$. The area under the curve in Fig. A.2, page A.11, between the depths d_1 and d_2 is the integral

$$A = \int_{d_1}^{d_2} \frac{d\ell}{dd}\,dd = \ell_2 - \ell_1$$

where $\ell_2 - \ell_1$ is the length of the channel between sections 1 and 2. Since $d\ell/dd$ is a function of d, which is not readily integrated, it will be easier to evaluate the area under the curve by numerical calculations. A close approximation of this area is the area of a parallelogram having parallel sides of lengths $2R_1/s_o$ and $2R_2/s_o$ and width of $(d_2 - d_1)$. The

area of this parallelogram is

$$A = \frac{1}{2}\left[\frac{2R_1}{s_o} + \frac{2R_2}{s_o}\right](d_2 - d_1) \equiv \frac{1}{s_o}(R_1 + R_2)(d_2 - d_1) \qquad \text{A.15}$$

which has been shown to be approximately equal to $\ell_2 - \ell_1$, the length of the channel between sections 1 and 2. The approximation is improved considerably by selecting smaller values of increments of depths $(d_2 - d_1)$.

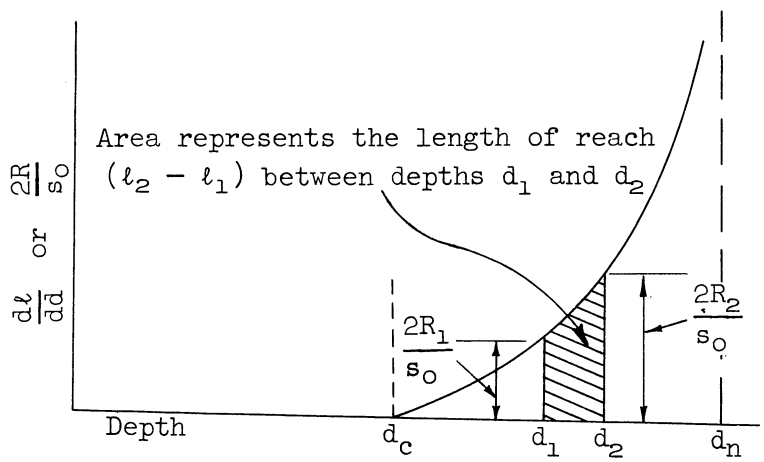


FIGURE A.2

In the derivation of the varied-flow equation, the origin of ℓ was considered as some point upstream from sections 1 and 2 and ℓ was positive in the direction of flow; further, section 1 was considered to be upstream from section 2. Hence, in all calculations, if section 1 is upstream from section 2, the quantity $(\ell_2 - \ell_1)$ must be positive irrespective of the direction in which computations are carried.

The evaluation of water-surface profiles for prismatic channels having horizontal and adverse bottom slopes may be solved in a similar way using Eqs. A.11 and A.13. See Exs. 2 and 3, pages A.17 and A.18.

Introduction to second method. The method of solving for a water-surface profile by the second method is distinctly different from that presented by the foregoing method. Water-surface profiles for prismatic channels would seldom be determined by this method, since the first method presented is simpler and more direct. This method would be advantageous for a given prismatic channel if a number of water-surface profiles are required for various discharges Q.

The following method is basically a re-evaluation and extension of Francis F. Escoffier's[1] method, in which changes in velocity heads were neglected. This method does not neglect velocity heads. A considerable amount

[1]Escoffier, Francis F., Graphical Calculation of Backwater Eliminates Solution by Trial, Engineering News-Record, June 27, 1946.

of computations may be eliminated by using certain relationships between the depth of flow and channel characteristics as shown by Bakhmeteff[1], and Von Seggern[2]. These relationships show a nearly linear variation of depth of flow d and the characteristics $s_o/Q^2_{n.d}$ and $Q_{c.d}$ when log-log plots are made. This is illustrated by Figs. 2, 3, and 4 of Ex. 4, ES-83, page A.19.

Non-prismatic channels, such as natural waterways, have cross sections given at definite stations. Thus, definite lengths of reaches are given and the depth of flow is the variable to be determined. Dictating that the depth be the variable to be determined complicates the solution for water-surface profiles because the depth is expressed implicitly in the equation.

Certain close approximations and assumptions will be made in the development of this method. Only by increasing the number of cross sections and correct evaluation of Manning's roughness coefficient n, rather than algebraic manipulation of formulas, is the error in predicting a water-surface profile kept small. The error introduced by the given approximations will, without exception, be smaller than that introduced by incorrect evaluation of n and an inaccurate and insufficient number of cross sections.

Once the water-surface profile has been solved for a given discharge in a particular channel, only a small amount of additional work is required to compute water-surface profiles for a large number of other discharges.

The type of problem solvable by the second method will have the following data given:

1. The discharge Q, for which the water-surface profile is determined.

2. The cross sections of the channel.

3. The distance between the given cross sections.

4. The slopes of the channel bottom or the elevations of the bottom of the given cross sections.

5. Manning's roughness coefficient n for various flow areas of the cross sections.

6. The depth of flow at some section or a control section. Generally, this depth will not be given but can be approximated closely. A method of making this approximation is given.

The first step in solving a water-surface profile, as in the foregoing method, is the determination of which breaks in grade in the channel bottom are control sections. See page 4, ES-38. When no depth of flow is given and the channel has no control section, the depth of flow can be closely approximated for subcritical flows in the following manner. Determine a

---

[1]Bakhmeteff, Boris A., Hydraulics of Open Channels, McGraw-Hill, Equation 70, page 84.

[2]Von Seggern, M. E., Integrating the Equation of Non-uniform Flow, ASCE Transactions, V. 115, (1950), page 71.

probable maximum and minimum depth at a sufficiently great distance downstream from the portion of the channel for which the water-surface profile is to be solved. Compute two water-surface profiles, in an upstream direction, commencing the profiles at these maximum and minimum depths. These two water-surface profiles converge to nearly the same depth of flow at the downstream section of the reach of channel for which the profiles are to be determined when the maximum and minimum depths have been chosen at a sufficiently great distance downstream. See Ex. 4, ES-83, page A.19.

Development of equation. By Bernoulli's theorem, see Fig. A.3.

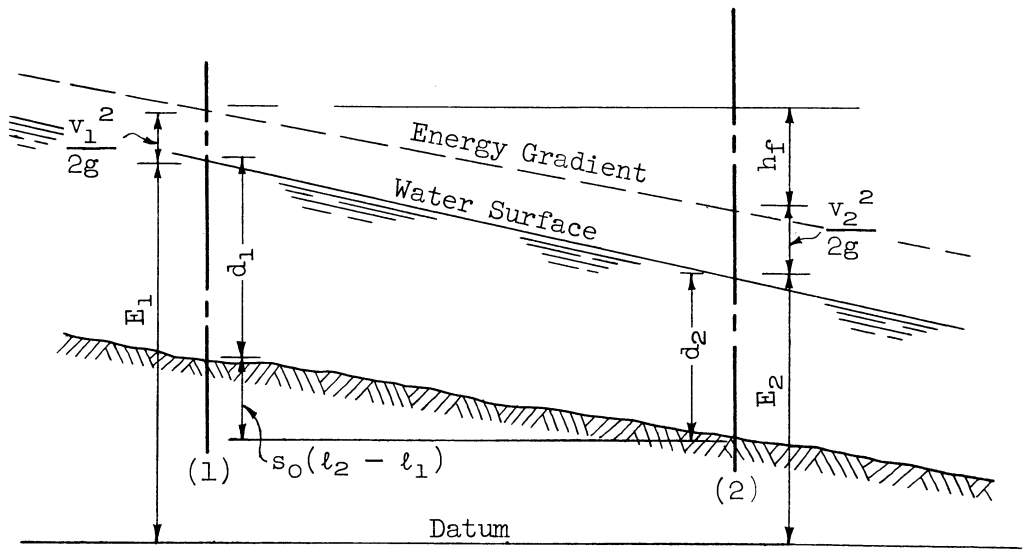


FIGURE A.3

$$E_1 + \frac{v_1^2}{2g} = E_2 + \frac{v_2^2}{2g} + h_f \qquad \text{A.16}$$

where E = elevation of water surface in feet
h_f = friction head loss between sections 1 and 2

Rewriting

$$E_1 - E_2 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + h_f$$

As shown by Eq. A.7, the rate of friction-head loss at any section 1, s_1, is

$$s_1 = s_o \left[\frac{Q}{Qn, d_1}\right]^2$$

Assuming the rate of friction-head loss s for the reach $(\ell_2 - \ell_1)$ as the average of the friction-head loss at section 1 and 2, or

$$h_f = \frac{s_o}{2}\left\{\left[\frac{Q}{Qn, d_1}\right]^2 + \left[\frac{Q}{Qn, d_2}\right]^2\right\}(\ell_2 - \ell_1) \qquad \text{A.17}$$

Observing that

$$v = \frac{Q}{a}$$

obtain on substituting into Eqs. A.16 and A.17

$$E_1 - E_2 = \left[\frac{Q^2}{2ga_2^2} - \frac{Q^2}{2ga_1^2}\right] + \frac{Q^2(\ell_2 - \ell_1)g}{2g}\left[\frac{s_o}{Q_{n,d_1}^2} + \frac{s_o}{Q_{n,d_2}^2}\right]$$

Rearranging

$$\frac{E_1 - E_2}{\left[\frac{1}{a_2^2} + \frac{s_o}{Q_{n,d_2}^2}(\ell_2 - \ell_1)g\right] - \left[\frac{1}{a_1^2} - \frac{s_o}{Q_{n,d_1}^2}(\ell_2 - \ell_1)g\right]} = \frac{Q^2}{2g} \qquad \text{A.18}$$

on letting

$$U_2^+ = \frac{1}{a_2^2} + (\ell_2 - \ell_1)g\,\frac{s_o}{Q_{n,d_2}^2} \qquad \text{A.19}$$

and

$$U_1^- = \frac{1}{a_1^2} - (\ell_2 - \ell_1)g\,\frac{s_o}{Q_{n,d_1}^2} \qquad \text{A.20}$$

Equation A.18 becomes

$$\frac{E_1 - E_2}{U_2^+ - U_1^-} = \frac{Q^2}{2g} \qquad \text{A.21}$$

This formula is the form used to determine water-surface profiles. Assume d_2 is given for the discharge along with the first 5 listed quantities of page A.12. If $d_2 > d_c$, then water-surface-profile calculations are made in an upstream direction and it is required that d_1 be determined. The terms E_1 and U_1^- are functions of d_1, and E_2 and U_2^+ are functions of d_2. Thus, every term in Eq. A.21 is known except E_1 and U_1^-. The terms E_1 and U_1^- would also be known if d_1 were known. Select various values of d_1 and plot the U_1^- curve, using ordinates of E_1 and abscissas of U_1^-. Plot the point P_2 having the coordinates U_2^+, E_2 for the given value of d_2. (See Fig. A.4, page A.15)

Draw a straight line having a slope of $-\frac{Q^2}{2g}$ from P_2 to intersect the curve U_1^- at point P_1. By this construction, the point P_1 has the relationship

$$\frac{E_1 - E_2}{U_2^+ - U_1^-} = \tan\theta = \frac{Q^2}{2g}$$

But this is also the relationship which must be satisfied to determine the water-surface profile, see Eq. A.21. Therefore, this is a graphical solution for the water-surface profile and E_1 at the point P_1 is the water-surface elevation at section 1. Moreover, this construction constitutes a

direct graphical solution of the depth of flow d_1. This graphical solution is of a form which requires little additional work to determine the water-surface profile for any discharge once the values of U_2^+ and U_1^- have been computed.

The procedure for computing a water-surface profile in natural channels is illustrated in Exs. 4, 5, and 6 of ES-83, pages A.19 through A.25. Much computational work can be eliminated in determining the values of $Q_{c,d}$, $\frac{s_o}{Q_{n,d}^2}$, $\frac{1}{a^2}$, U_2^+, and U_1^- for intermediate values of depths by observing the nearly straight-line relationship which exists on a log-log plot when the side slopes of the banks remain nearly constant. See ES-83, Figs. 3, 4, 5, 6, and 7, pages A.19 and A.20. Coordinates for these plots should be determined at depths of flows at which the cross-sectional bank slope abruptly changes. If no appreciable change in the slope of banks occurs in the region of depths under consideration, these values should be evaluated at depth intervals of 3 to 4 feet. The plots of Figs. 2 and 3, page A.19, are used to determine which sections are control sections, and whether flow is subcritical or supercritical.

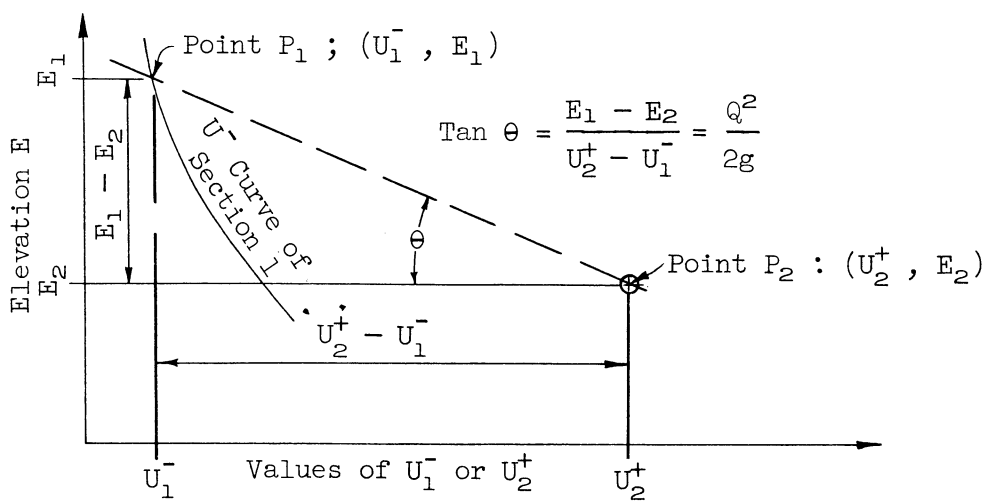
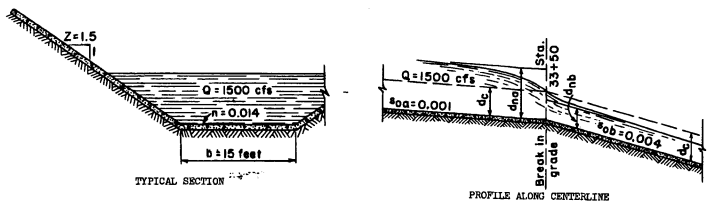


FIGURE A.4

HYDRAULICS: NON-UNIFORM FLOW IN A PRISMATIC CHANNEL WITH POSITIVE BOTTOM SLOPE —— Example 1.

Given: A concrete trapezoidal channel with the values of z = 1.5, b = 15 ft, n = 0.014, Q = 1500 cfs. The stationing is in the direction of flow. A break in grade is located at Sta 33 + 50 and elevation of 100.0; the slope upstream from Sta 33+50, s_{oa}, is 0.0010. The slope downstream from Sta 33+50, s_{ob}, is 0.004



Determine: The water surface profile

Solution: Rewrite Eq. A.8 in the form

$$s_o \frac{dt}{dd} = \frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_c d}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3} s_o^{1/2}}}{\frac{nQ_n d}{b^{8/3} s_o^{1/2}}}\right]^2 - 1} = 2R$$

1. Solve whether the break in grade is a control section

a. Solve for d_c corresponding to Q = 1500

$$\frac{Q}{b} = \frac{1500}{15} = 100 \text{ cfs/ft}; \quad \frac{z}{b} = \frac{1.5}{15} = 0.1$$

From ES-24 d_c = 5.59 ft

b. Solve for critical slope s_c corresponding to Q = 1500 cfs, d_c = 5.59 ft

$$\frac{d_c}{b} = \frac{5.59}{15} = 0.3727$$

From ES-55 $\dfrac{nQ}{b^{8/3} s_c^{1/2}} = 0.341$ where $\dfrac{nQ}{b^{8/3}} = \dfrac{(0.014)(1500)}{15^{8/3}} = 0.01533$

$$s_c = \left[\frac{nQ}{b^{8/3}}\right]^2 \frac{1}{(0.341)^2} = \left[\frac{0.01533}{0.341}\right]^2 = 0.002022$$

Hence, Sta 33 + 50 is a control section since $s_{oa} < s_c < s_{ob}$

2. Solve for the normal depth of flow in the upstream and downstream reaches.

a. For the upstream reach

$$\frac{nQ}{b^{8/3} s_{oa}^{1/2}} = \frac{0.01533}{s_{oa}^{1/2}} = \frac{0.01533}{(0.001)^{1/2}} = 0.4850$$

From ES-55 $\dfrac{d_{na}}{b} = 0.449$ and d_{na} = 15 (0.449) = 6.73 ft

b. For the downstream reach

$$\frac{nQ}{b^{8/3} s_{ob}^{1/2}} = \frac{0.01533}{(0.004)^{1/2}} = 0.2425$$

From ES-55 $\dfrac{d_{nb}}{b} = 0.31$ and d_{nb} = 15 (0.31) = 4.65 ft

3. Solve for the water-surface profile and tabulate values in Table 1.

a. column 1 lists arbitrarily selected values of depth between d_c and d_{na} or d_c and d_{nb}
b. column 3 is read from ES-55
c. column 4 is read from ES-24
d. column 5 is the value of $\dfrac{nQ}{b^{8/3} s_o^{1/2}}$ = 0.4850 or 0.2425 divided by column 3 (See step 2 above)
e. column 6 is the value Q/b = 100 divided by column 4
f. column 7 is read from ES-53 using columns 5 and 6
g. column 10 is obtained by the relation $t_2 - t_1 = \dfrac{1}{s_o}(R_1 + R_2)(d_2 - d_1)$ · Eq. A.15

The value of R versus depth d is plotted in Fig. 1. The area under the curve between any two depths represents the length of reach between these depths. This plot is useful for ascertaining whether or not the selected depths in column 1 have been chosen at sufficiently close intervals to justify the use of the relation shown by Eq. A.15.

The profile can be readily plotted by using columns 1 and 11.

TABLE 1

Direction	(1) d	(2) $\frac{d}{b}$	(3) $\frac{nQ_{n,d}}{b^{8/3}s_o^{1/2}}$	(4) $\frac{Q_{c,d}}{b}$	(5) $\frac{Q}{Q_{n,d}}$	(6) $\frac{Q}{Q_{c,d}}$	(7) R	(8) $R_1 + R_2$	(9) $d_2 - d_1$	(10) $t_2 - t_1$	(11) Station
Upstream from break in grade, s_{oa} = 0.001	5.59	0.373	0.341	100.0	1.422	1.000	0	- 0.0177	-0.05	0.885	33+50
	5.64	0.376	0.347	101.7	1.398	0.983	- 0.0177	- 0.0591	-0.06	3.55	33+49.1
	5.70	0.380	0.354	103.8	1.370	0.963	- 0.0414	- 0.1246	-0.10	12.46	33+45.6
	5.80	0.387	0.366	107.0	1.325	0.935	- 0.0832	- 0.2212	-0.10	22.12	33+33.1
	5.90	0.393	0.377	110.5	1.286	0.905	- 0.138	- 0.346	-0.10	34.60	33+11.0
	6.00	0.400	0.390	113.8	1.244	0.879	- 0.208	- 0.506	-0.10	50.60	32+76.4
	6.10	0.407	0.402	117.1	1.206	0.854	- 0.298	- 0.725	-0.10	72.50	32+25.8
	6.20	0.413	0.415	120.6	1.169	0.829	- 0.427	- 1.045	-0.10	104.50	31+53.3
	6.30	0.420	0.428	124.1	1.133	0.806	- 0.618	- 1.569	-0.10	156.90	30+48.8
	6.40	0.427	0.442	127.7	1.097	0.783	- 0.951	- 2.441	-0.10	244.10	28+91.9
	6.50	0.433	0.454	131.2	1.068	0.762	- 1.49	- 4.33	-0.10	433.00	26+47.8
	6.60	0.440	0.467	134.9	1.039	0.741	- 2.84	- ∞	-0.13	∞	22+14.8
	6.73	0.449	0.485	140.2	1.000	0.713	- ∞				
Downstream from break in grade, s_{ob} = 0.004	5.59	0.373	0.341	100.0	0.711	1.000	0	- 0.0591	-0.09	1.33	33+50
	5.50	0.367	0.331	97.4	0.733	1.027	- 0.0591	- 0.2091	-0.10	5.23	33+51.3
	5.40	0.360	0.320	94.2	0.758	1.062	- 0.150	- 0.406	-0.10	10.15	33+56.6
	5.30	0.353	0.308	91.5	0.787	1.093	- 0.256	- 0.660	-0.10	16.50	33+66.7
	5.20	0.347	0.299	88.5	0.811	1.130	- 0.404	- 1.046	-0.10	26.15	33+83.2
	5.10	0.340	0.288	85.3	0.842	1.172	- 0.642	- 1.653	-0.10	41.33	34+09.4
	5.00	0.333	0.277	82.4	0.875	1.214	- 1.011	- 2.631	-0.10	65.78	34+50.7
	4.90	0.327	0.267	79.8	0.908	1.253	- 1.62	- 4.79	-0.10	119.75	35+16.5
	4.80	0.320	0.257	76.9	0.944	1.300	- 3.17	-14.70	-0.10	367.50	36+36.2
	4.70	0.313	0.247	74.1	0.982	1.350	-11.53	- ∞	-0.05	∞	40+03.7
	4.65	0.310	0.242	72.7	1.000	1.376	- ∞				

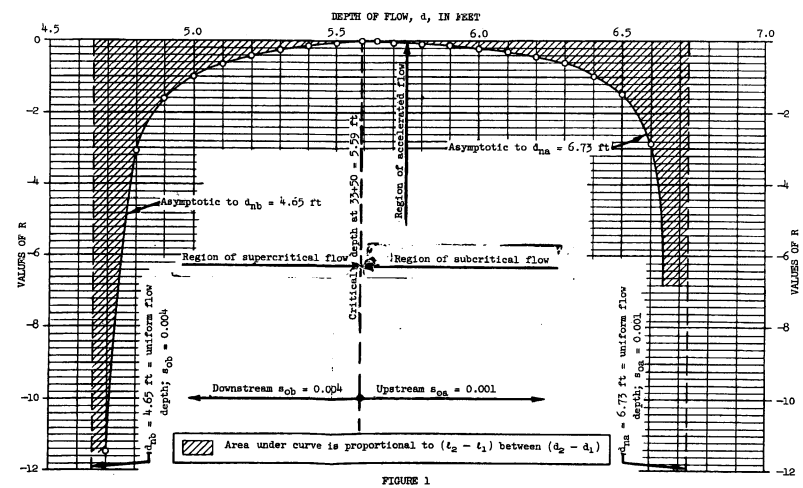


FIGURE 1

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO.
ES – 83
SHEET 1 OF 10
DATE 8-6-54

HYDRAULICS: NON-UNIFORM FLOW IN A PRISMATIC CHANNEL WITH A LEVEL BOTTOM — Example 2

EXAMPLE 2

Given: A prismatic trapezoidal earth spillway with 3 to 1 side slopes. The bottom of the spillway is level and is 200 ft long and 75 ft wide in the reach between the control section and the reservoir. Manning's coefficient of roughness n, of the spillway within this reach, is estimated to be 0.035. The spillway is expected to convey 1500 cfs.

Determine: A. The water-surface profile in the spillway between the reservoir and the control section.
B. The friction loss through this spillway measured in feet.
C. The elevation of the water surface in the reservoir when the spillway is discharging 1500 cfs.

Solution: Rewrite Eq. A.11 in the form

$$\frac{d_t}{d_d} = \frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3}s_o^{1/2}}}{\frac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}}\right]^2} = R_o$$

where $Q_{n,d}$ is defined as the normal discharge corresponding to a depth d and channel bottom slope of $s_o = 1.0$. Equation A.11 is to be used only for channels with horizontal bottoms. The quantity s_o by definition is unity when working with Eq. A.11 and Manning's formula even though the actual slope of the channel is zero.

A.1. Solve for the depth of flow at the control section Sta 2+00. The depth of flow at the control section is critical depth corresponding to the discharge Q = 1500 cfs.

$$\frac{Q}{b} = \frac{1500}{75} = 20 \text{ cfs/ft}; \qquad \frac{z}{b} = \frac{3}{75} = 0.04$$

From ES-24, $d_c = 2.25$ ft

2. Solve for the value of $\dfrac{nQ}{b^{8/3}s_o^{1/2}}$

$$\frac{nQ}{b^{8/3}s_o^{1/2}} = \frac{(0.035)(1500)}{(75)^{8/3}(1)^{1/2}} = 5.2480 \times 10^{-4}$$

3. Prepare Table 1
   (a) Column 1 lists arbitrarily selected depths greater than critical depth, beginning with $d_c = 2.25$ ft. The range of required depths cannot be readily predetermined.
   (b) Column 2 lists the selected depths of column 1 divided by the bottom width of the channel.
   (c) Column 3 is read from ES-55.
   (d) Column 4 is read from ES-24.
   (e) Column 5 lists $\dfrac{nQ}{b^{8/3}s_o^{1/2}} = 5.2480 \times 10^{-4}$ (see step 2 above) divided by column 3.
   (f) Column 6 lists Q/b = 20 divided by column 4.
   (g) Columns 7 and 8 are the squares of columns 5 and 6.
   (h) Column 9 lists column 8 minus unity. This is the numerator of the right-hand member of Eq. A.11.
   (i) Column 10 lists column 9 divided by column 7. This is equal to the left-hand member of Eq. A.11.
   (j) Column 11 lists the sum of the values of R_o at the end sections of each reach.
   (k) Column 12 lists the average value of R_o for each reach or column 11 times $\frac{1}{2}$.
   (l) Column 13 lists the difference in selected depth of column 1.
   (m) Column 14 lists the length of each reach between the selected depths listed in column 1 and is column 12 times column 13.
   (n) Column 15 lists the distances from the control section in the upstream direction and is the accumulated total of column 14.

B.1. The specific energy at the control section is

$$H_{ec} = d_c + \frac{v_c^2}{2g}$$

$$a_c = d_c(b + zd_c) = 2.25\left[75 + 3(2.25)\right] = 183.94 \text{ ft}^2$$

$$v_c = \frac{Q}{a_c} = \frac{1500}{183.94} = 8.155 \text{ ft/sec}$$

$$\frac{v_c^2}{2g} = \frac{(8.155)^2}{64.32} = 1.034 \text{ ft} \qquad H_{ec} = 2.25 + 1.034 = 3.284 \text{ ft}$$

2. The specific energy at Sta 0+00 is

$$H_{eo} = d_o + \frac{v_o^2}{2g}$$

$d_o = 3.77$ ft (by interpolating between d = 3.7 and d = 3.8 and observing that the control section is 200 ft downstream from the reservoir pool.)

$$a_o = d_o(b + zd_o) = 3.77\left[75 + 3(3.77)\right] = 325.39 \text{ ft}^2$$

$$v_o = \frac{Q}{a_o} = \frac{1500}{325.39} = 4.610 \text{ ft/sec}$$

$$\frac{v_o^2}{2g} = \frac{(4.610)^2}{64.32} = 0.3304 \text{ ft}$$

$$H_{eo} = 3.77 + 0.330 = 4.100 \text{ ft}$$

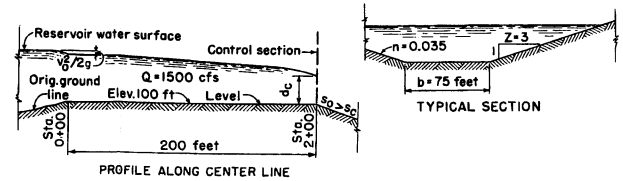
3. Since the spillway bottom is level, the difference in the specific energy head at Sta 0+00 and the control section $(H_{eo} - H_{ec})$ is the friction head loss h_f in this reach.

$$h_f = H_{eo} - H_{ec} = 4.100 - 3.284 = 0.816 \text{ ft}$$

C.1. The elevation of the water surface in the reservoir is equal to the specific energy head at Sta 0+00 plus the elevation of the bottom of channel at Sta 0+00. Elevation of water surface in the reservoir is

$$100.0 + H_{eo} = 104.100 \text{ ft}$$

The trapezoidal shape of this spillway does not exist for the full depth of flow in the reach near the entrance of the spillway. By neglecting the effect of this condition, as is done in the example, the water surface elevation in the reservoir required to produce a given discharge through the spillway is slightly greater than the elevation required had this effect been evaluated.



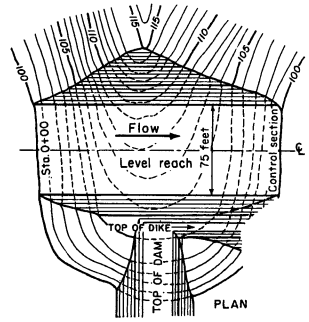


TABLE 1

1 d	2 $\frac{d}{b}$	3 $\frac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}$	4 $\frac{Q_{c,d}}{b}$	5 $\frac{Q}{Q_{n,d}}$	6 $\frac{Q}{Q_{c,d}}$	7 $\left[\frac{Q}{Q_{n,d}}\right]^2$	8 $\left[\frac{Q}{Q_{c,d}}\right]^2$	9 $\left[\frac{Q}{Q_{c,d}}\right]^2 - 1$	10 R_o	11 $R_{o1} + R_{o2}$	12 $\frac{R_{o1} + R_{o2}}{2}$	13 $d_2 - d_1$	14 $\ell_2 - \ell_1$	15 Distance from Control Section
2.25	0.03000	0.00442	20.00	0.11873	1.0000	0.014097	1.00000	0	0					0
										− 5.423	− 2.712	−0.05	0.136	
2.3	0.03067	0.00459	20.75	0.11434	0.9639	0.013074	0.92910	−0.07090	− 5.423					0.136
										− 21.328	− 10.664	−0.10	1.066	
2.4	0.03200	0.00492	22.10	0.10667	0.9050	0.011378	0.81903	−0.18097	− 15.905					1.202
										− 44.317	− 22.158	−0.10	2.216	
2.5	0.03333	0.00527	23.60	0.09958	0.8475	0.0099162	0.71826	−0.28174	− 28.412					3.418
										− 70.175	− 35.087	−0.10	3.509	
2.6	0.03467	0.00564	25.03	0.09305	0.7990	0.0086583	0.63840	−0.36160	− 41.763					6.927
										− 99.053	− 49.527	−0.10	4.953	
2.7	0.03600	0.00604	26.55	0.08689	0.7533	0.0075499	0.56746	−0.43254	− 57.290					11.880
										−130.638	− 65.319	−0.10	6.532	
2.8	0.03733	0.00641	28.05	0.08187	0.7130	0.0067027	0.50837	−0.49163	− 73.348					18.412
										−164.848	− 82.424	−0.10	8.242	
2.9	0.03867	0.00680	29.65	0.07718	0.6745	0.0059568	0.45495	−0.54505	− 91.500					26.654
										−203.171	−101.585	−0.10	10.159	
3.0	0.04000	0.00721	31.30	0.07279	0.6390	0.0052984	0.40832	−0.59168	−111.671					36.813
										−245.401	−122.701	−0.10	12.270	
3.1	0.04133	0.00763	33.00	0.06878	0.6061	0.0047307	0.36736	−0.63264	−133.730					49.083
										−290.428	−145.214	−0.10	14.521	
3.2	0.04267	0.00805	34.60	0.06519	0.5780	0.0042497	0.33408	−0.66592	−156.698					63.604
										−338.776	−169.388	−0.10	16.939	
3.3	0.04400	0.00849	36.25	0.06181	0.5517	0.0038205	0.30437	−0.69563	−182.078					80.543
										−392.743	−196.384	−0.10	19.638	
3.4	0.04533	0.00895	38.10	0.05864	0.5249	0.0034396	0.27552	−0.72448	−210.663					100.181
										−449.333	−224.679	−0.10	22.468	
3.5	0.04667	0.00937	39.90	0.05601	0.5013	0.0031371	0.25130	−0.74870	−238.668					122.649
										−510.256	−255.128	−0.10	25.513	
3.6	0.04800	0.00986	41.65	0.05323	0.4802	0.0028334	0.23059	−0.76941	−271.588					148.162
										−578.583	−289.292	−0.10	28.929	
3.7	0.04933	0.01035	43.60	0.05071	0.4587	0.0025720	0.21041	−0.78959	−306.995					177.091
										−648.693	−324.347	−0.10	32.435	
3.8	0.05067	0.01080	45.50	0.04859	0.4396	0.0023610	0.19325	−0.80675	−341.698					209.526

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO.
ES-83
SHEET 2 OF 10
DATE 7-22-54

HYDRAULICS: NON-UNIFORM FLOW IN A PRISMATIC CHANNEL WITH A NEGATIVE BOTTOM SLOPE — EXAMPLE 3

EXAMPLE 3

Given: A prismatic earth spillway with side slopes of 3 to 1. The spillway has a level bottom upstream from the control section for a distance of 90 ft and an adverse slope of 10 to 1 for an upstream distance of 100 ft. The bottom of the spillway is 75 ft wide. Manning's coefficient of roughness n is estimated as 0.035 feet.

Determine: A. The water-surface profile in the spillway when the discharge is 1500 cfs.

B. The friction loss from Sta 0+00 to Sta 1+90 and the elevation of the water surface in the reservoir when the spillway is discharging 1500 cfs.

C. The friction loss through the spillway from Sta 0+00 to Sta 1+00.

Solution: Rewrite Eq. A.13 in the form

$$|s_o| \frac{dt}{dd} = \frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3}|s_o|^{1/2}}}{\frac{n\,Q_{n,d}}{b^{8/3}|s_o|^{1/2}}}\right]^2 + 1} = R_n$$

A.1. Solve for the critical depth of flow corresponding to the discharge of 1500 cfs.

$$\frac{Q}{b} = \frac{1500}{75} = 20 \text{ cfs/ft}$$

From ES-24, $d_c = 2.25$ ft. This is the depth of flow at the control section.

2. Solve for the depth of flow at the break in grade between the 10 to 1 negative slope and the level reach. This was illustrated by Ex. 2 and found to be 3.347 ft.

3. Solve for the value of

$$\frac{nQ}{b^{8/3}|s_o|^{1/2}} = \frac{(0.035)(1500)}{(75)^{8/3}(0.10)^{1/2}} = 1.6596 \times 10^{-3}$$

4. Solve the water-surface profile upstream from the break in grade at Sta 1+00 starting with a depth of 3.347 ft and tabulate values in table 1.

(a) Column 1 lists arbitrarily selected values of depths between the depth of flow at Sta 1+00 and an approximated depth at Sta 0+00. The approximated depth at Sta 0+00 is estimated by assuming the water-surface elevation at Sta 0+00 to be slightly greater than the elevation of the energy gradient at Sta 1+00. The specific energy at Sta 1+00 is

$$H_{e1} = d_1 + \frac{v_1^2}{2g}$$

$$a_1 = d_1(b + zd_1) = 3.347\left[75 + 3(3.347)\right] = 284.63 \text{ ft}^2$$

$$v_1 = \frac{Q}{a_1} = \frac{1500}{284.63} = 5.27 \text{ ft/sec}$$

$$\frac{v_1^2}{2g} = \frac{(5.27)^2}{64.32} = 0.4318 \text{ ft}$$

$$H_{e1} = 3.347 + 0.4318 = 3.779 \text{ ft}$$

The elevation of the energy gradient at Sta 1+00 is

$$(100.00 + H_{e1}) = 103.779 \text{ ft}$$

The approximated depth of flow at Sta 0+00 is the difference in the elevation of the energy gradient at Sta 1+00 and bottom of channel at Sta 0+00.

$$103.779 - 90 = 13.779 \text{ ft}$$

Try 14 ft for an upper limit of d in column 1.

(b) Column 3 is read from ES-55.

(c) Column 4 is read from ES-24.

(d) Column 5 is the value of $\frac{nQ}{b^{8/3}|s_o|^{1/2}} = 1.6596 \times 10^{-3}$ (see step 3) divided by column 3.

(e) Column 6 is the value of Q/b = 20 divided by column 4.

(f) Columns 7 and 8 are the squares of columns 5 and 6.

(g) Column 9 lists the values of R_n as given by Eq. A.13 or is

$$R_n = \frac{\text{column 8 minus unity}}{\text{column 7 plus unity}}$$

(h) Column 10 lists the average value of R_n for each reach.

(i) Column 11 lists the difference in depth of flow at the end section of each reach.

(j) Column 12 lists column 11 divided by $|s_o|$.

(k) Column 13 lists the length of each reach and is column 10 times column 12.

(l) Column 14 lists the distance upstream from the break in grade at Sta 1+00 and is the accumulated total of column 13.

(m) Column 15 lists the stationing for the selected depths of flow used in column 1.

B.1. By Ex. 2, the specific energy at Sta 1+90 is $H_{ec} = 3.284$ ft.

2. The specific energy at Sta 0+00 is

$$H_{eo} = d_o + \frac{v_o^2}{2g} = 13.824 \text{ ft}$$

$d_o = 13.81$ ft by interpolation between d = 13 ft and d = 14 ft

3. The friction loss from Sta 0+00 to Sta 1+90 is the difference in the elevation of the energy gradient at Sta 0+00 to Sta 1+90. The elevation of the energy gradient at Sta 0+00 is the elevation of the bottom of the spillway plus the specific energy at Sta 0+00.

$$90.00 + H_{eo} = 103.824 \text{ ft}$$

The elevation of the energy gradient at Sta 1+90 is

$$100.00 + H_{ec} = 103.284$$

The friction loss is

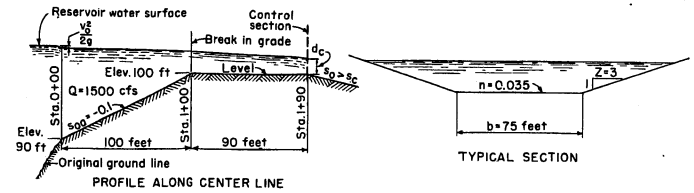
$$103.824 - 103.284 = 0.540 \text{ ft}$$

The elevation of the water surface in the reservoir is equal to the elevation of the energy gradient at Sta 0+00 or 103.824 ft.

C.1. The friction loss from Sta 0+00 to Sta 1+00 is the difference in the elevation of the energy gradient. By A.4, the elevation of the energy gradient at Sta 1+00 is 103.779 ft. By B.3, the elevation of the energy gradient is 103.824 ft. The friction loss between Sta 0+00 and Sta 1+00 is

$$103.824 - 103.779 = 0.045 \text{ ft}$$

The trapezoidal shape of this spillway does not exist for the full depth of flow in the reach near the entrance of the spillway. By neglecting the effect of this condition, as is done in the example, the water surface elevation in the reservoir required to produce a given discharge through the spillway is slightly greater than the elevation required had this effect been evaluated.



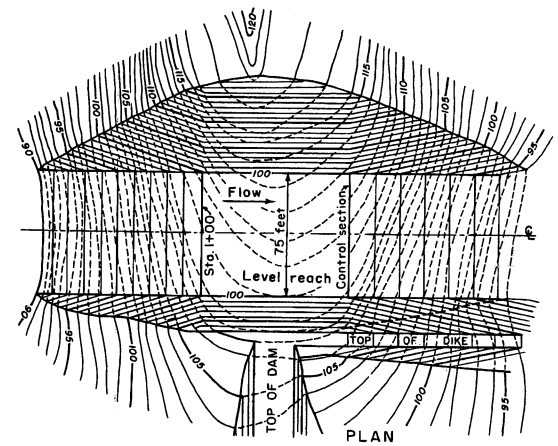


TABLE 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d	$\frac{d}{b}$	$\frac{n\,Q_{n,d}}{b^{8/3}\lvert s_o\rvert^{1/2}}$	$\frac{Q_{c,d}}{b}$	$\frac{Q}{Q_{n,d}}$	$\frac{Q}{Q_{c,d}}$	$\left[\frac{Q}{Q_{n,d}}\right]^2$	$\left[\frac{Q}{Q_{c,d}}\right]^2$	R_n	$\frac{R_{n1}+R_{n2}}{2}$	d_2-d_1	$\frac{(d_2-d_1)}{\lvert s_o\rvert}$	t_2-t_1	$\Sigma(t_2-t_1)$	Station
3.347	0.04463	0.00875	37.10	0.1897	0.5391	0.03599	0.2906	−0.6848					0	1+00.00
									−0.7049	−0.153	−1.53	1.078		
3.50	0.04667	0.00942	39.80	0.1762	0.5025	0.03105	0.2525	−0.7250					1.078	0+98.92
									−0.7729	−0.50	−5.00	3.865		
4.00	0.05333	0.01186	49.50	0.1399	0.4040	0.01957	0.1632	−0.8207					4.943	0+95.06
									−0.8487	−0.50	−5.00	4.244		
4.50	0.06000	0.01456	59.75	0.1140	0.3347	0.01300	0.1120	−0.8766					9.187	0+90.81
									−0.8940	−0.50	−5.00	4.470		
5.00	0.06667	0.01752	70.50	0.09473	0.2837	0.008974	0.08049	−0.9113					13.66	0+86.34
									−0.9312	−1.00	−10.0	9.312		
6.00	0.08000	0.02410	94.95	0.06886	0.2106	0.004742	0.04435	−0.9511					22.97	0+77.03
									−0.9608	−1.00	−10.0	9.608		
7.00	0.09333	0.03175	121.9	0.05227	0.1641	0.002732	0.02693	−0.9704					32.58	0+67.42
									−0.9757	−1.00	−10.0	9.757		
8.00	0.1067	0.0402	152.1	0.04128	0.1315	0.001704	0.01729	−0.9810					42.33	0+57.67
									−0.9842	−1.00	−10.0	9.842		
9.00	0.1200	0.04974	185.4	0.03337	0.1079	0.001114	0.01164	−0.9873					52.18	0+47.82
									−0.9892	−1.00	−10.0	9.892		
10.0	0.1333	0.06025	221.4	0.02755	0.09033	0.0007590	0.008160	−0.9911					62.07	0+37.93
									−0.9924	−1.00	−10.0	9.924		
11.0	0.1467	0.07210	260.7	0.02302	0.07672	0.0005299	0.005886	−0.9936					71.99	0+28.01
									−0.9945	−1.00	−10.0	9.945		
12.0	0.1600	0.08460	303.2	0.01962	0.06596	0.0003849	0.004351	−0.9953					81.94	0+18.06
									−0.9959	−1.00	−10.0	9.959		
13.0	0.1733	0.09799	348.7	0.01694	0.05736	0.0002870	0.003290	−0.9964					91.90	0+ 8.10
									−0.9969	−1.00	−10.0	9.969		
14.0	0.1867	0.1142	397.5	0.01453	0.05031	0.0002111	0.002531	−0.9973					101.9	

HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL —Example 4.

THIS EXAMPLE ILLUSTRATES A METHOD TO DETERMINE THE APPROXIMATE DEPTH OF FLOW AT A SECTION

Generally channel cross sections for a natural channel are determined by field surveys at definite stations. The length of the reach between two consecutive sections is therefore fixed or given and a depth of flow is to be determined. In Example 1 the length was determined between two given (actually selected) depths. The relationship between the depth of flow and any hydraulic characteristics of a cross section is the same for every cross section in a prismatic channel. This enables one to choose the length of the reach as the variable to be determined, and the length of the reach between two given depths is determined in Example 1. Since length of reach is given in natural channels the depth of flow is the variable to be determined. This complicates the solution for water-surface profiles because the depth of flow is implicity expressed in the differential equation.

Any method of water-surface profile determination requires that at least one depth of flow corresponding to the discharge, Q, under consideration be given at a station or be determinable. The determination of a depth of flow in a natural channel at a control section presents a trivial problem, for the depth of flow at the section is known to be the critical depth. Some channels for which water-surface profiles are to be determined have no control section, nor is the depth of flow given for any portion of the channel. This example is concerned with the determination of a depth of flow at station 0+00 so water-surface profile computations can be made upstream from station 0+00.

Given: Channel cross sections for a natural channel have been determined by field surveys at stations indicated by Fig. 1. Stationing is in a downstream direction. The roughness coefficient, n, has been estimated to be 0.035 for in-bank flow and 0.06 for out-bank flow. Dikes having 3 to 1 slope have been constructed as shown in Fig. 1. These dikes are sufficiently high to contain the flow of 7000 cfs. The depth of flow at station 17+90 has been determined to be not less than 14 ft nor greater than 18 ft for a discharge of 7000 cfs.

Determine: The approximate depth of flow at station 0+00 when the discharge is 7000 cfs.

Solution: Equation A.18 is used for this solution.

$$\frac{E_1 - E_2}{\left[\frac{1}{a_2^2} + \frac{s_o}{Q_{n,d_2}^2}(\ell_2 - \ell_1)g\right] - \left[\frac{1}{a_1^2} - \frac{s_o}{Q_{n,d_1}^2}(\ell_2 - \ell_1)g\right]} = \frac{Q^2}{2g}$$

1. Prepare tabular form given by Table 1.
   (a) Column 1 lists the stations of cross sections and elevation of channel bottom at these stations.
   (b) Column 2 is an arbitrary selection of flow depths for an estimated range of depth of flow. A selection of depths should be made at each major change in cross section shape and not over 3 or 4 ft interval between consecutive depths.
   (c) Columns 3 and 7 list the flow areas associated with the appropriate n values as obtained from Fig. 1.
   (d) Columns 4 and 8 list the wetted perimeters associated with the appropriate n values as obtained from Fig. 1.
   (e) Columns 5 and 9 list the cross section factor F read from ES-76.
   (f) Columns 6 and 10 are read from ES-77.
   (g) Column 11 is the sum of columns 6 and 10.
   (h) Column 12 is the square of the reciprocal of column 11. This can be read from the double scale of ES-77.

2. Prepare Figures 2 and 3 from Table 1. These values are plotted on log-log coordinate paper. Figure 2 will be used to determine critical slope. Figure 3 will be used later to determine values of $s_o + Q_{n,d}^2$ for intermediate values of depths of flow.

3. Solve which sections are control sections for the discharge of 7000 cfs.
   (a) Prepare tabular form as given by Table 2.
      (i) Columns 1 and 2 are the same as columns 1 and 2 of Table 1
      (ii) Column 3 lists the top width corresponding to the depth in column 2
      (iii) Column 4 lists the total area corresponding to the depth in column 2
      (iv) Column 5 is read from ES-75
   (b) Prepare Figure 4 on log-log coordinate paper. Values are obtained from Table 2.
   (c) Prepare tabular form as given by Table 3.
      (i) Column 2 will be considered later
      (ii) Column 4 is read from Figure 4 on line Q = 7000 cfs and are critical depths d_c for Q = 7000 cfs
      (iii) Column 5 is read from Figure 2 for critical depths, d_c, in column 4
      (iv) Column 6 lists $s_c^{1/2}$ which is 7000 cfs divided by column 5
      (v) Column 8 lists the slope of channel bottom

No control sections exist for a discharge of 7000 cfs for all bottom slopes, s_o, are less than critical slope, s_c. Thus, flow is subcritical and computations will be carried in an upstream direction.

4. Solve for approximate depth of flow at station 0+00.
   (a) Prepare tabular form given by Table 4.
      (i) Column 2 lists the same range of depths as Table 1 at closer intervals
      (ii) Prepare Figure 5 on log-log coordinate paper from Table 2. Observe that the right hand scale is the square of the reciprocal of the left hand scale in ES-77 and may be used in determining $1 + a^2$ values
      (iii) Column 3 is read from Figure 5
      (iv) Column 4 is read from Figure 3
      (v) Column 5 lists the product of column 4 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of the downstream reach from the station under consideration. The station under consideration is section 1 as is indicated by the subscript d_1 of Q_{n,d_1} in this column heading.
      (vi) Column 6 is the product of column 4 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of the upstream reach from the station under consideration. The station under consideration is section 2 as is indicated by the subscript d_2 of Q_{n,d_2} in this column heading.
      (vii) Column 7 lists column 3 minus column 5
      (viii) Column 8 lists column 3 plus column 6
   (b) It may be desirable to prepare Figures 6 and 7, particularly if values of d have not been selected sufficiently close in column 2.
   (c) Prepare Figure 8.
      (i) Plot U_1^- vs elevation curves
      (ii) Plot U_2^+ vs elevation curves
      (iii) Column 2 of Table 3 gives the slope of straight lines connecting the curves U_1^- and U_2^+. The slope scale for Figure 8 is determined by dividing the ordinate above the reference point by the distance between the scale and the reference point. In this example the reference point has been arbitrarily taken at elevation 1185 and an abscissa of 2.0×10^{-6} from the slope scale. The value of the slope scale at the line having an elevation of 1186 is $(1186 - 1185) \div 2.0 \times 10^{-6} = 5 \times 10^5$.
      (iv) Make the graphical solution for the water-surface profile if the depth of flow at station 17+90 is 14 ft. The beginning point from which a straight line is drawn having the slope given by column 2, Table 3, is at depth of 14 ft or elevation 1184.2 on U_2^+ curve. The elevation of flow at station 13+30 is the intersection of this straight line with the U_1^- curve. The elevation of flow is now known at station 13+30 and the elevation of flow at 9+10 is determined in a similar manner. This procedure is continued to station 0+00. The depth obtained at 0+00 is the depth of flow if the depth of flow at 17+90 is 14 ft.

      A graphical solution for the water-surface profile if the depth of flow at station 17+90 is 18 ft is obtained in a similar manner. Observe the depth of flow at station 0+00 is not less than 13.61 ft nor greater than 14.22 ft for it has been given the depth of flow at 17+90 is not less than 14 ft nor greater than 18 ft. Thus, the depth of flow at station 0+00 has been closely approximated. If a closer approximation is desired it will be necessary to determine water-surface profiles from a station downstream from station 17+90
      (v) Figure 9 shows water-surface profiles determined for various depths at station 17+90.

TABLE 1

1	2	3	4	5	6	7	8	9	10	11	12
Station and Bottom Elevation	d ft	a ft²	p ft	F	$\frac{Q_{n,d}}{s_o^{1/2}}$	a ft²	p ft	F	$\frac{Q_{n,d}}{s_o^{1/2}}$	$\frac{Q_{n,d}}{s_o^{1/2}}$	$\frac{s_o}{Q_{n,d}^2}$
	n = 0.035					n = 0.06				composite n	
17+90	9.6	384	64.9	1867	53,300	15.5	38.2	12.65	211	53,511	3.485×10^{-10}
	11.6	508	64.9	2985	85,300	103.1	50.8	246.0	4,100	89,400	1.250×10^{-10}
1170.2	13.6	632	64.9	4284	122,400	214.7	63.5	720.0	12,000	134,400	5.550×10^{-11}
	15.6	756	64.9	5772	164,900	350.3	76.1	1440.0	24,000	188,900	2.806×10^{-11}
	17.6	880	64.9	7435	212,400	515.0	88.7	2480.0	41,330	253,730	1.555×10^{-11}
13+30	9.4	376	61.2	1872	53,500	15.0	40.2	11.50	1,920	53,692	3.475×10^{-10}
	11.4	492	61.2	2930	83,700	107.0	52.8	254.4	4,240	87,940	1.296×10^{-10}
1171.6	13.4	608	61.2	4180	119,400	223.0	65.5	750.0	12,500	131,900	5.750×10^{-11}
	15.4	724	61.2	5585	159,600	363.0	78.2	1500.0	25,000	184,600	2.950×10^{-11}
	17.4	840	61.2	7155	204,400	527.0	90.8	2530.0	42,180	246,580	1.644×10^{-11}
9+10	10.2	439	69.9	2221	63,500	23.6	52.2	20.6	343	63,843	2.453×10^{-10}
	12.2	571	69.9	3439	98,300	139.6	64.8	346.0	5,767	104,067	9.220×10^{-11}
1173.6	14.2	703	69.9	4865	139,000	279.6	77.4	978.7	16,310	155,310	4.140×10^{-11}
	16.2	835	69.9	6495	185,600	443.6	90.1	1908.0	31,800	217,400	2.118×10^{-11}
4+50	12.4	623	89.3	3380	96,600	21.4	30.6	25.05	418	97,018	1.062×10^{-10}
	14.4	795	89.3	5070	144,900	94.4	43.3	236.0	3,933	148,833	4.525×10^{-11}
1174.8	16.4	967	89.3	7030	200,900	191.4	56.0	646.0	10,770	211,670	2.235×10^{-11}
0+00	13.4	615	83.7	3455	98,700	15.4	28.9	15.04	251	98,951	1.022×10^{-10}
	14.4	694	83.7	4225	120,700	47.4	35.2	85.90	1,432	122,132	6.690×10^{-11}
1175.8	15.4	773	83.7	5060	144,600	85.4	41.6	204.8	3,413	148,013	4.560×10^{-11}
	16.4	852	83.7	5950	170,000	129.4	47.9	374.0	6,233	176,233	3.220×10^{-11}
	17.4	931	83.7	6890	196,900	179.4	54.3	591.5	9,858	206,758	2.350×10^{-11}

TABLE 2

1	2	3	4	5
Station and Bottom Elevation	Depth ft	T ft	a ft²	$Q_{c,d}$ cfs
0+00	13.4	108.2	630	8,610
1175.8	14.4	114.2	741	10,730
	15.4	120.2	858	13,100
	16.4	126.2	981	15,540
	17.4	132.2	1110	18,250
4+50	12.4	116.5	644	8,630
1174.8	14.4	128.5	889	13,280
	16.4	140.5	1158	18,850
9+10	10.2	118.0	463	5,200
1173.6	12.2	130.0	711	9,450
	14.2	142.0	983	14,700
	16.2	154.0	1279	20,910
13+30	9.4	98.0	391	4,410
1171.6	11.4	110.0	599	7,925
	13.4	122.0	831	12,310
	15.4	134.0	1087	17,570
	17.4	146.0	1367	23,800
17+90	9.6	99.8	399	4,535
1170.2	11.6	111.8	611	8,110
	13.6	123.8	847	12,630
	15.6	135.8	1106	17,930
	17.6	147.8	1395	24,400

TABLE 3

1	2	3	4	5	6	7	8	
Q	$\frac{Q^2}{2g}$	Sta	d_c	$\frac{Q_{n,d_c}}{s_c^{1/2}}$	$s_c^{1/2}$	s_c	s_o	Remarks
7000	7.618×10^5	0+00	12.53*	81,000	0.08642	0.007468	0.00222	$s_c > s_o$
		4+50	11.50*	78,100	0.08963	0.008034	0.00261	$s_c > s_o$
		9+10	11.14	81,500	0.08589	0.007377	0.00476	$s_c > s_o$
		13+30	10.95	79,600	0.08794	0.007733	0.00504	$s_c > s_o$
		17+90	11.05	78,500	0.08917	0.007951		

*These values were obtained by extrapolation from Figure 4.

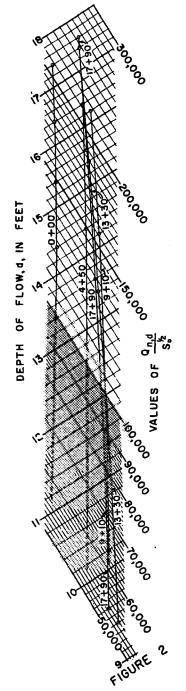


FIGURE 2

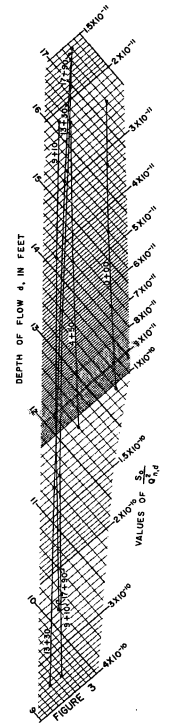


FIGURE 3

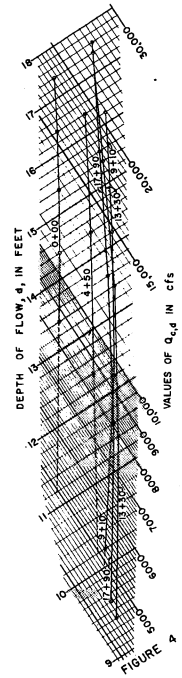


FIGURE 4

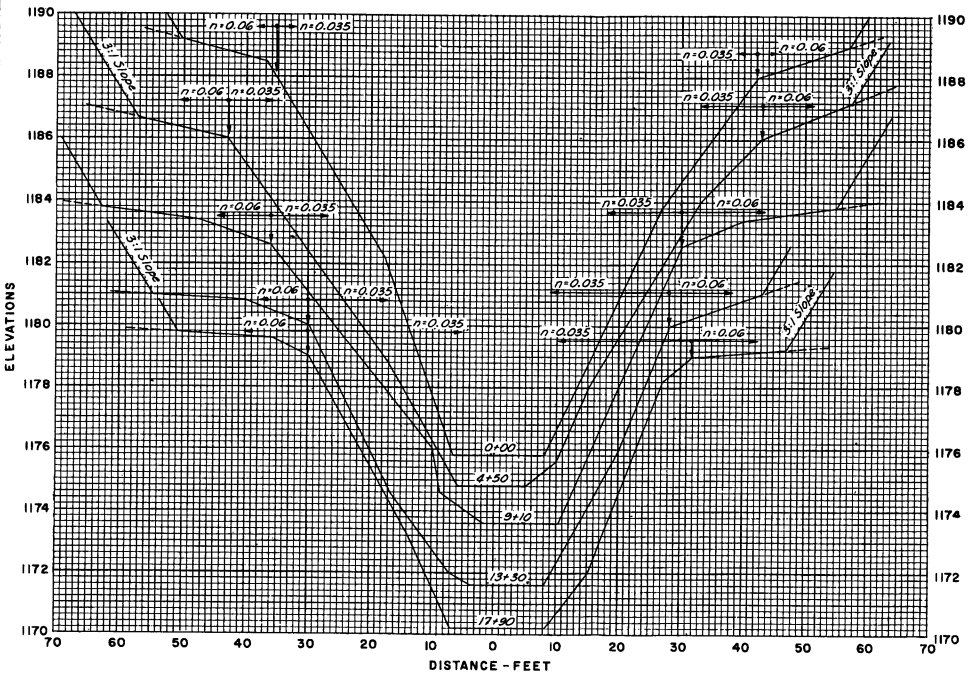
REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDAD DWG. NO. ES-83
SHEET 4 OF 10
DATE 7-23-54

HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL — Example 4.

FIGURE 1 — GIVEN CROSS SECTIONS



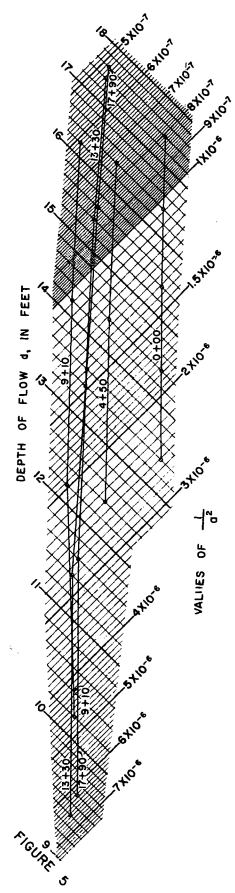


FIGURE 5

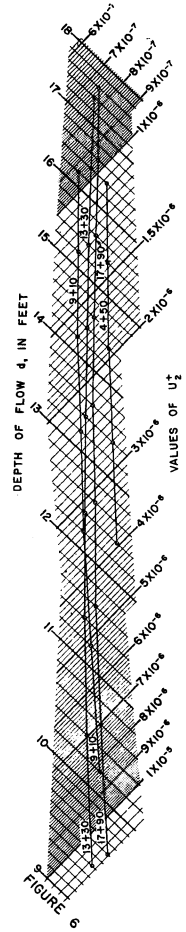


FIGURE 6

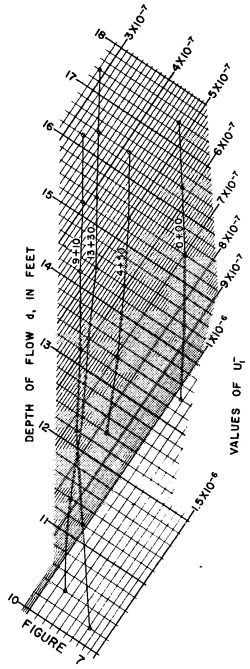


FIGURE 7

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-83
SHEET 5 OF 10
DATE 4-22-54

HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL — Example 4

TABLE 4

1	2	3	4	5	6	7	8	9
Station and Elevation	d ft	$\frac{1}{a^2}$	$\frac{s_o}{Q^2_{n,d}}$	$\frac{s_o(\ell_2-\ell_1)g}{Q^2_{n,d_1}}$	$\frac{s_o(\ell_2-\ell_1)g}{Q^2_{n,d_2}}$	U_1^-	U_2^+	E
17+90	9.6	6.260×10^{-8}	3.485×10^{-10}		5.156×10^{-8}		11.42×10^{-8}	1179.8
	10.6	4.010×10^{-8}	2.045×10^{-10}		3.025×10^{-8}		7.035×10^{-8}	1180.8
1170.2	11.6	2.678×10^{-8}	1.250×10^{-10}		1.849×10^{-8}		4.527×10^{-8}	1181.8
	12.6	1.915×10^{-8}	8.210×10^{-11}		1.215×10^{-8}		3.130×10^{-8}	1182.8
	13.6	1.393×10^{-8}	5.530×10^{-11}		0.8181×10^{-8}		2.213×10^{-8}	1183.8
	14.6	1.060×10^{-8}	3.920×10^{-11}		0.5799×10^{-8}		1.640×10^{-8}	1184.8
	15.6	0.8170×10^{-8}	2.806×10^{-11}		0.4151×10^{-8}		1.232×10^{-8}	1185.8
	16.6	0.6480×10^{-8}	2.065×10^{-11}		0.3084×10^{-8}		0.9565×10^{-8}	1186.8
	17.6	0.5130×10^{-8}	1.553×10^{-11}		0.2297×10^{-8}		0.7427×10^{-8}	1187.8
13+30	9.4	6.540×10^{-8}	3.475×10^{-10}	5.141×10^{-8}	4.694×10^{-8}	1.399×10^{-8}	11.23×10^{-8}	1181.0
	10.4	4.180×10^{-8}	2.080×10^{-10}	3.077×10^{-8}	2.809×10^{-8}	1.103×10^{-8}	6.990×10^{-8}	1182.0
1171.6	11.4	2.785×10^{-8}	1.296×10^{-10}	1.917×10^{-8}	1.750×10^{-8}	0.8677×10^{-8}	4.536×10^{-8}	1183.0
	12.4	2.000×10^{-8}	8.620×10^{-11}	1.275×10^{-8}	1.164×10^{-8}	0.7248×10^{-8}	3.164×10^{-8}	1184.0
	13.4	1.447×10^{-8}	5.750×10^{-11}	0.8506×10^{-8}	0.7767×10^{-8}	0.5964×10^{-8}	2.224×10^{-8}	1185.0
	14.4	1.105×10^{-8}	4.085×10^{-11}	0.6045×10^{-8}	0.5518×10^{-8}	0.5007×10^{-8}	1.657×10^{-8}	1186.0
	15.4	0.8450×10^{-8}	2.932×10^{-11}	0.4337×10^{-8}	0.3960×10^{-8}	0.4112×10^{-8}	1.241×10^{-8}	1187.0
	16.4	0.6680×10^{-8}	2.185×10^{-11}	0.3232×10^{-8}	0.2951×10^{-8}	0.3448×10^{-8}	0.9651×10^{-8}	1188.0
	17.4	0.5340×10^{-8}	1.644×10^{-11}	0.2432×10^{-8}	0.2221×10^{-8}	0.2908×10^{-8}	0.7561×10^{-8}	1189.0
9+10	10.2	4.670×10^{-8}	2.453×10^{-10}	3.313×10^{-8}	3.629×10^{-8}	1.357×10^{-8}	8.299×10^{-8}	1183.8
	11.2	2.975×10^{-8}	1.475×10^{-10}	1.992×10^{-8}	2.182×10^{-8}	0.9827×10^{-8}	5.157×10^{-8}	1184.8
1173.6	12.2	1.975×10^{-8}	9.220×10^{-11}	1.245×10^{-8}	1.364×10^{-8}	0.7296×10^{-8}	3.339×10^{-8}	1185.8
	13.2	1.415×10^{-8}	6.100×10^{-11}	0.8259×10^{-8}	0.9024×10^{-8}	0.5911×10^{-8}	2.317×10^{-8}	1186.8
	14.2	1.036×10^{-8}	4.140×10^{-11}	0.5592×10^{-8}	0.6125×10^{-8}	0.4768×10^{-8}	1.649×10^{-8}	1187.8
	15.2	0.7910×10^{-8}	2.950×10^{-11}	0.3985×10^{-8}	0.4364×10^{-8}	0.3925×10^{-8}	1.227×10^{-8}	1188.8
	16.2	0.6120×10^{-8}	2.118×10^{-11}	0.2861×10^{-8}	0.3133×10^{-8}	0.3259×10^{-8}	0.9253×10^{-8}	1189.8
4+50	12.4	2.410×10^{-8}	1.062×10^{-10}	1.571×10^{-8}	1.537×10^{-8}	0.8389×10^{-8}	3.947×10^{-8}	1187.2
	13.4	1.725×10^{-8}	6.860×10^{-11}	1.015×10^{-8}	0.9928×10^{-8}	0.7102×10^{-8}	2.718×10^{-8}	1188.2
1174.8	14.4	1.264×10^{-8}	4.525×10^{-11}	0.6694×10^{-8}	0.6549×10^{-8}	0.5946×10^{-8}	1.919×10^{-8}	1189.2
	15.4	0.9670×10^{-8}	3.160×10^{-11}	0.4675×10^{-8}	0.4573×10^{-8}	0.4995×10^{-8}	1.424×10^{-8}	1190.2
	16.4	0.7470×10^{-8}	2.235×10^{-11}	0.3306×10^{-8}	0.3234×10^{-8}	0.4164×10^{-8}	1.071×10^{-8}	1191.2
0+00	13.4	2.518×10^{-8}	1.022×10^{-10}	1.479×10^{-8}		1.0390×10^{-8}		1189.2
	14.4	1.822×10^{-8}	6.690×10^{-11}	0.9682×10^{-8}		0.8538×10^{-8}		1190.2
1175.8	15.4	1.360×10^{-8}	4.560×10^{-11}	0.6599×10^{-8}		0.7001×10^{-8}		1191.2
	16.4	1.038×10^{-8}	3.220×10^{-11}	0.4660×10^{-8}		0.5720×10^{-8}		1192.2
	17.4	0.8100×10^{-8}	2.350×10^{-11}	0.3401×10^{-8}		0.4699×10^{-8}		1193.2

FIGURE 8

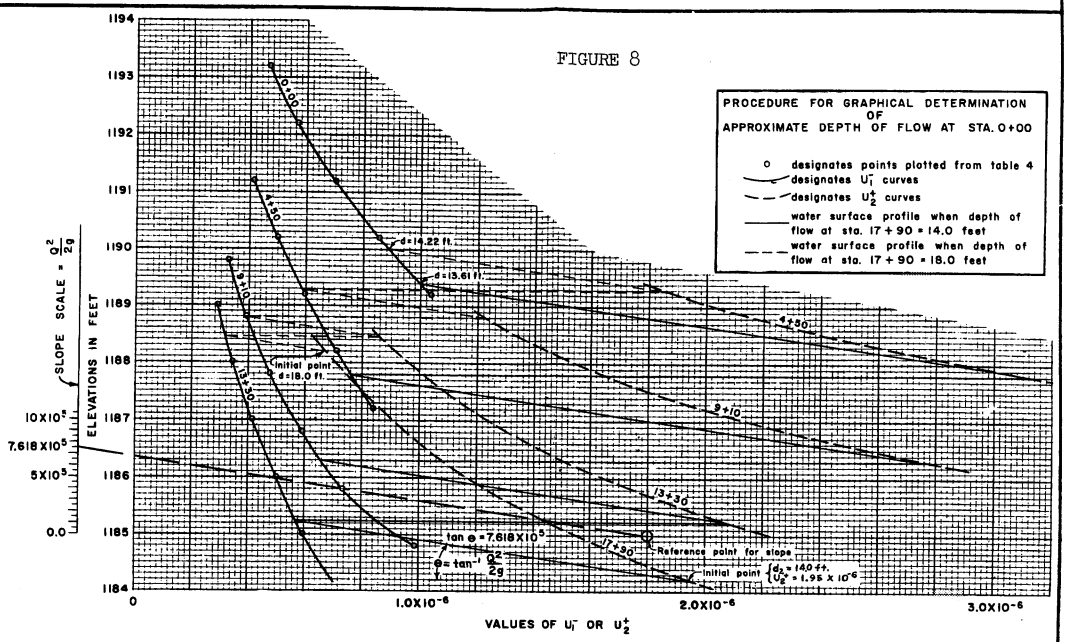
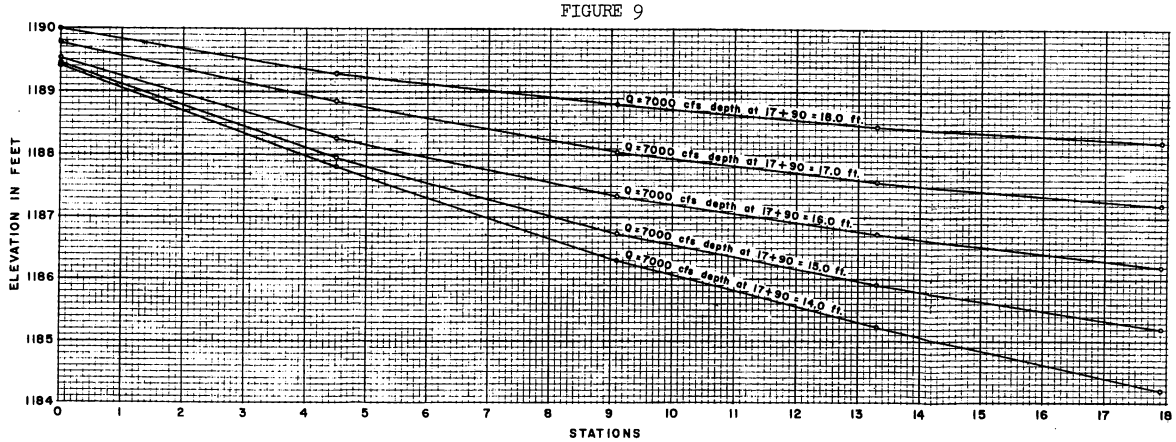


FIGURE 9



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-83
SHEET 6 OF 10
DATE 4-22-54

HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL FOR VARIOUS DISCHARGES

Example 5

EXAMPLE 5

This example deals with the same natural channel as given in Ex. 4 merely to eliminate repetition of an additional set of similar data. Example 5 is distinctly different from Ex. 4 and the problems are in no way related. The approximate depth of flow was determined at Sta 0+00 in Ex. 4. In Ex. 5 the depths at Sta 17+90 have been determined and water-surface profiles are to be determined for various discharges.

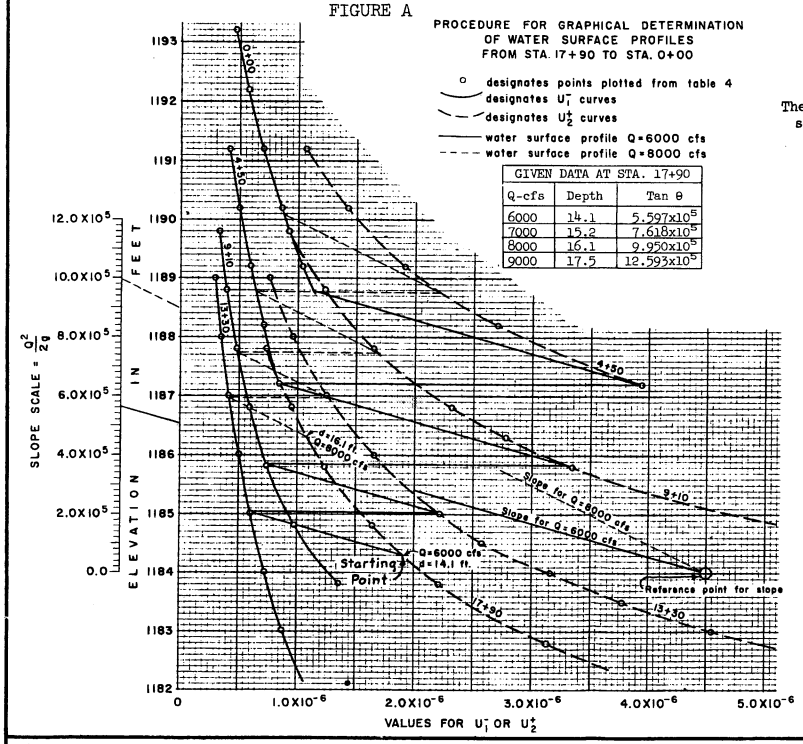
Given: Channel cross sections and stationing of a natural channel along with roughness coefficient n as shown by Fig. 1 of Ex. 4. The dikes are sufficiently high to contain flows of 9000 cfs. The depths of flow at Sta 17+90 have been determined for various discharges.

Determine: The water-surface profiles for discharges 6000, 7000, 8000, and 9000 cfs and the discharge vs depth curve at Sta 0+00.

Discharge cfs	Given depth of flow at sta. 17+90 ft
6000	14.1
7000	15.2
8000	16.1
9000	17.5

FIGURE A

PROCEDURE FOR GRAPHICAL DETERMINATION OF WATER SURFACE PROFILES FROM STA. 17+90 TO STA. 0+00

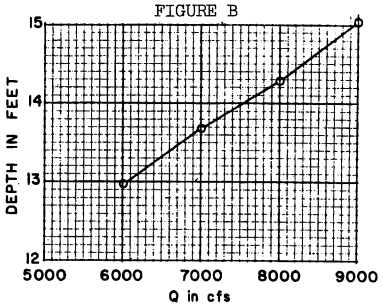


Solution: Equation A.18 is used for this solution. (See Ex. 4)

1. Prepare tabular form given by Table 1, Ex. 4.
2. Prepare Figs. 2 and 3 of Ex. 4.
3. Solve which sections are control sections for discharges of 6000, 7000, 8000, and 9000 cfs. This procedure is described by Ex. 4 and the results are given in Table A. No control sections exist for all discharges considered since all bottom slopes s_o are less than critical slope s_c.
4. Solve for water-surface profiles corresponding to the given discharges.
   a. Prepare tabular form given by Table 4, Ex. 4.
   b. It may be desirable to prepare Figs. 6 and 7 of Ex. 4.
   c. Prepare Fig. A similar to Fig. 8 of Ex. 4.
      (i) Plot U_1^- vs elevation curves.
      (ii) Plot U_2^+ vs elevation curves.
      (iii) Column 2 of Table A gives the slopes of straight lines connecting the curves U_1^- and U_2^+ for the various discharges. This procedure is explained in Ex. 4, item 4c, (iii)
      (iv) Make graphical solution for water surface profiles corresponding to the given discharges. and depths of flow at Sta 17+90. This procedure has been described in Ex. 4.

The graphical solution for the discharges Q = 6000 and 8000 cfs are shown. The discharge-depth curve at Sta 0+00 is given by Fig. B.

FIGURE B



RATING CURVE AT STA. 0+00

TABLE A

1	2	3	4	5	6	7	8	9
Q	$\frac{Q^2}{2g}$	Station	d_c	$\frac{Q_n d_c}{s_c^{1/2}}$	$s_c^{1/2}$	s_c	s_o	Remarks
6000	5.597×10^5	0+00	11.92*	69,000	0.08696	0.007562	.00222	$s_c > s_o$
		4+50	10.90*	67,000	0.08955	0.008019	.00261	$s_c > s_o$
		9+10	10.65	72,000	0.08333	0.006944	.00476	$s_c > s_o$
		13+30	10.40	69,700	0.08608	0.007410	.00304	$s_c > s_o$
		17+90	10.52	68,800	0.08721	0.007606		
7000	7.618×10^5	0+00	12.55*	81,000	0.08642	0.007468	.00222	$s_c > s_o$
		4+50	11.50*	78,100	0.08963	0.008034	.00261	$s_c > s_o$
		9+10	11.14	81,500	0.08589	0.007377	.00476	$s_c > s_o$
		13+30	10.95	79,600	0.08794	0.007733	.00304	$s_c > s_o$
		17+90	11.05	78,500	0.08917	0.007951		
8000	9.950×10^5	0+00	13.10*	92,000	0.08696	0.007562	.00222	$s_c > s_o$
		4+50	12.05*	90,000	0.08889	0.007901	.00261	$s_c > s_o$
		9+10	11.60	91,000	0.08791	0.007728	.00476	$s_c > s_o$
		13+30	11.44	88,900	0.08999	0.008098	.00304	$s_c > s_o$
		17+90	11.55	88,200	0.09070	0.008226		
9000	12.59×10^5	0+00	13.60	103,000	0.08738	0.007635	.00222	$s_c > s_o$
		4+50	12.56	101,000	0.08911	0.007941	.00261	$s_c > s_o$
		9+10	12.02	101,000	0.08911	0.007941	.00476	$s_c > s_o$
		13+30	11.92	98,000	0.09184	0.008435	.00304	$s_c > s_o$
		17+90	12.05	98,750	0.09114	0.008306		

*These values were obtained by extrapolation from Figure 4.

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION -DESIGN SECTION

STANDARD DWG. NO. ES- 83
SHEET 7 OF 10
DATE: 5-13-54

HYDRAULICS: NON-UNIFORM FLOW IN A NON-PRISMATIC CHANNEL WITH LEVEL AND ADVERSE BOTTOM SLOPES
Example 6.

EXAMPLE 6

Given: A trapezoidal earth spillway 190 ft long from the reservoir to the control section. Stationing is in a downstream direction and Sta 0+00 is located at the reservoir. The bottom of the spillway at Sta 0+00 is 95 ft wide and converges uniformly to a width of 75 ft at Sta 1+00. The bottom width of 75 ft is constant from Sta 1+00 to Sta 1+90. The bottom slope of the spillway between Sta 0+00 and Sta 1+00 is adverse and is -3% between Sta 0+00 and Sta 0+80 and +20% between Sta 0+80 and Sta 1+00. The bottom of the spillway is level between Sta 1+00 and Sta 1+90. Manning's coefficient of roughness n is estimated to be 0.035.

Determine: A. The water-surface profiles for discharges of 600, 800, 1000, 1200, 1500, and 1800 cfs in this spillway.
B. Curve showing discharge vs elevation of water surface in the reservoir.
C. Friction loss in spillway between Sta 0+00 and Sta 1+90 for Q = 1500 cfs.

Solution: Equation A.18 is used for this solution.

$$\frac{E_1 - E_2}{\left[\frac{1}{a_2^2} + \frac{s_o}{Q_{n,d_2}^2}(\ell_2 - \ell_1)g\right] - \left[\frac{1}{a_1^2} - \frac{s_o}{Q_{n,d_1}^2}(\ell_2 - \ell_1)g\right]} = \frac{Q^2}{2g}$$

Stations 0+00, 0+50, 0+80, 0+90, 1+00, 1+50, 1+70, 1+80, and 1+90 have been arbitrarily selected as those stations at which depths of flow are to be determined.

A.1. Prepare tabular form given by Table 1.
- (a) Column 1 lists the selected stations at which depths of flow are to be determined and the elevations of the channel bottom at these stations.
- (b) Column 2 is an arbitrary selection of flow depths for an estimated range of depth of flow.
- (c) Column 3 lists the bottom widths of the channel at the selected stations in column 1.
- (d) Column 4 lists the cross-sectional areas for the depths in column 2.
- (e) Column 5 lists the wetted perimeters for the depths in column 2.
- (f) Column 6 is read from ES-76.
- (g) Column 7 is the square of the reciprocal of column 4. This can be read from the double scale of ES-77.
- (h) Column 8 is read from ES-77.
- (i) Column 9 lists the product of Column 8 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of reach downstream from the station under consideration. The station under consideration is section 1 as is indicated by the subscript d_1 of Q_{n,d_1} in this column heading.
- (j) Column 10 lists the product of column 8 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of reach upstream from the station under consideration. The station under consideration is section 2 as is indicated by the subscript d_2 of Q_{n,d_2} in this column heading.
- (k) Column 11 lists values of U_1^- when the section under consideration is the upstream section or section 1. Column 11 lists column 7 minus column 9. (See Eq. A.20)
- (l) Column 12 lists values of U_2^+ when the section under consideration is the downstream section of the reach or section 2. Column 12 lists column 7 plus column 10. (See Eq. A.19)
- (m) Column 13 lists the elevations of the water surfaces corresponding to the depths of column 2 or the depths of column 2 added to the bottom elevations of column 1.

2. Prepare Figs. 1 and 2 by plotting column 2 vs columns 11 and 12 respectively on log-log paper.
3. Solve for the values of $Q^2/2g$ as shown by Table 2 for the discharges under consideration.
4. Solve for water-surface profiles corresponding to the given discharges.
   - (a) Prepare Figs. 3a and 3b.
     - (i) Plot U_1^- vs elevation curves for various values of d shown in Fig. 1.
     - (ii) Plot U_2^+ vs elevation curves for various values of d shown in Fig. 2.
     - (iii) Column 2 of Table 2 gives the slopes of straight lines connecting the U_1^- and U_2^+ curves for the various discharges. This procedure is explained in Ex. 4, item 4c, (iii).
     - (iv) Make graphical solution for water-surface profiles corresponding to the given discharges at Sta 0+00. This procedure has been described in Ex. 4, item 4c, (iv).

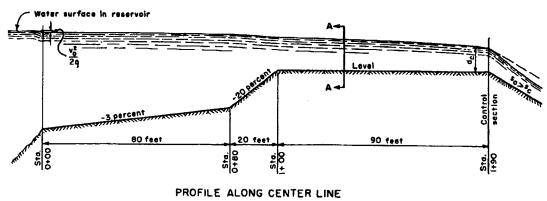
The graphical solutions for discharges 800 and 1500 have been shown on Figs. 3a and 3b respectively. Two figures, 3a and 3b, are drawn instead of a single figure because of the desirability of a scale adjustment. The water-surface profiles for the various discharges are given by Fig. 4. The rapid drop in elevation in water-surface profiles between Sta 0+80 and Sta 1+00 is the result of a change from potential head to kinetic head.

Observe that the depth of flow at Sta 1+00 for a discharge Q = 1500 cfs is 3.37 ft. Example 2 for this same channel and discharge shows a depth of flow of 3.347 ft at a section 90 ft upstream from the break in grade. The principal cause for the difference in the answers for the depth at the section 90 ft upstream from the control section by the two methods is the number of sections used upstream from the control section in computing this depth of flow. The result d = 3.347 ft, obtained in Ex. 2, is by far the more accurate because eleven intermediate sections were used in computing this depth, while in Ex. 6 only three intermediate sections were used in computing the depth d = 3.37 ft.

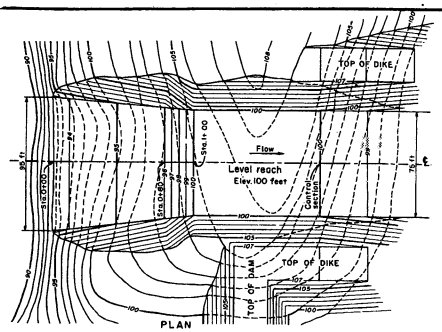
B.1. The depths of flow at Sta 0+00 for various discharges are given by Table 2. These values were read from Figs. 3a and 3b. The water-surface elevation in the reservoir is greater than the water-surface elevation at Sta 0+00 by the quantity $v_o^2/2g$ where v_o is the velocity in ft/sec at Sta 0+00. The curve showing discharge vs elevation of water surface in the reservoir is shown by Fig. 5.

C.1. The friction loss between Sta 0+00 and Sta 1+90 is the difference in elevation of the energy gradient. The elevation of the energy gradient at Sta 0+00 is 103.831 ft when Q = 1500 cfs. See Table 2. The elevation of the energy gradient at Sta 1+90 is 103.284 ft. See Ex. 2. The friction loss is 103.831 - 103.284 = 0.547 ft.

The trapezoidal shape of this spillway does not exist for the full depth of flow in the reach near the entrance of the spillway. By neglecting the effect of this condition, as is done in the example, the water-surface elevation in the reservoir required to produce a given discharge through the spillway is slightly greater than the elevation required had this effect been evaluated.



PROFILE ALONG CENTER LINE



SECTION A—A

TABLE 1

1	2	3	4	5	6	7	8	9	10	11	12	13
Station and Bottom Elevation	d ft	b ft	a ft²	P ft	F	$\frac{1}{a^2}$	$\frac{s_o}{Q_{n,d}^2}$	$\frac{s_o(\ell_2-\ell_1)g}{Q_{n,d_1}^2}$	$\frac{s_o(\ell_2-\ell_1)g}{Q_{n,d_2}^2}$	U_1^-	U_2^+	E
1+90	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸		14.6971x10⁻⁶		118.197x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹		2.0358x10⁻⁶		31.636x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹		0.7542x10⁻⁶		16.514x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰		0.1872x10⁻⁶		6.492x10⁻⁶	104.50
1+80	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸	14.6971x10⁻⁶	14.6971x10⁻⁶	88.803x10⁻⁶	118.197x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹	2.0357x10⁻⁶	2.0358x10⁻⁶	27.564x10⁻⁶	31.636x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹	0.7542x10⁻⁶	0.7542x10⁻⁶	15.006x10⁻⁶	16.514x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰	0.1872x10⁻⁶	0.1872x10⁻⁶	6.118x10⁻⁶	6.492x10⁻⁶	104.50
1+70	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸	14.6971x10⁻⁶	29.3942x10⁻⁶	88.803x10⁻⁶	132.894x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹	2.0357x10⁻⁶	4.0715x10⁻⁶	27.564x10⁻⁶	33.672x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹	0.7542x10⁻⁶	1.5083x10⁻⁶	15.006x10⁻⁶	17.268x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰	0.1872x10⁻⁶	0.3743x10⁻⁶	6.118x10⁻⁶	6.679x10⁻⁶	104.50
1+50	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸	29.3942x10⁻⁶	73.4856x10⁻⁶	74.106x10⁻⁶	176.986x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹	4.0715x10⁻⁶	10.1786x10⁻⁶	25.528x10⁻⁶	39.779x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹	1.5083x10⁻⁶	3.7708x10⁻⁶	14.252x10⁻⁶	19.531x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰	0.3743x10⁻⁶	0.9359x10⁻⁶	5.931x10⁻⁶	7.241x10⁻⁶	104.50
1+00	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸	73.4856x10⁻⁶	14.6971x10⁻⁶	30.014x10⁻⁶	118.197x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹	10.1786x10⁻⁶	2.0358x10⁻⁶	19.421x10⁻⁶	31.636x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹	3.7708x10⁻⁶	0.7542x10⁻⁶	11.989x10⁻⁶	16.514x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰	0.9359x10⁻⁶	0.1872x10⁻⁶	5.369x10⁻⁶	6.492x10⁻⁶	104.50
0+90	3.00	77.0	258.00	95.97	740.0	15.023x10⁻⁶	2.235x10⁻⁹	0.7188x10⁻⁶	0.7188x10⁻⁶	14.304x10⁻⁶	15.742x10⁻⁶	101.00
	4.50	77.0	407.25	105.46	1480.0	6.029x10⁻⁶	5.620x10⁻¹⁰	0.1807x10⁻⁶	0.1807x10⁻⁶	5.848x10⁻⁶	6.210x10⁻⁶	102.50
98.00	6.50	77.0	627.25	118.11	2825.0	2.542x10⁻⁶	1.536x10⁻¹⁰	0.04939x10⁻⁶	0.04939x10⁻⁶	2.493x10⁻⁶	2.591x10⁻⁶	104.50
0+80	4.00	79.0	364.00	104.29	1245.0	7.547x10⁻⁶	7.895x10⁻¹⁰	0.2539x10⁻⁶	0.7616x10⁻⁶	7.293x10⁻⁶	8.309x10⁻⁶	101.00
	6.50	79.0	640.25	120.11	2908.0	2.440x10⁻⁶	1.447x10⁻¹⁰	0.04653x10⁻⁶	0.1396x10⁻⁶	2.393x10⁻⁶	2.580x10⁻⁶	102.50
96.00	8.50	79.0	888.25	132.76	4675.0	1.267x10⁻⁶	5.590x10⁻¹¹	0.01797x10⁻⁶	0.05395x10⁻⁶	1.249x10⁻⁶	1.321x10⁻⁶	104.50
0+50	6.00	85.0	618.00	122.95	2690.0	2.618x10⁻⁶	1.695x10⁻¹⁰	0.1633x10⁻⁶	0.2722x10⁻⁶	2.455x10⁻⁶	2.890x10⁻⁶	101.10
	8.50	85.0	939.25	138.76	4985.0	1.134x10⁻⁶	4.925x10⁻¹¹	0.04752x10⁻⁶	0.07919x10⁻⁶	1.087x10⁻⁶	1.213x10⁻⁶	103.60
95.10	11.00	85.0	1298.00	154.57	7980.0	0.5935x10⁻⁶	1.940x10⁻¹¹	0.01872x10⁻⁶	0.03120x10⁻⁶	0.5748x10⁻⁶	0.6247x10⁻⁶	106.10
0+00	6.00	95.0	678.00	132.95	2980.0	2.175x10⁻⁶	1.380x10⁻¹⁰	0.2219x10⁻⁶		1.953x10⁻⁶		99.60
	8.50	95.0	1024.25	148.76	5520.0	0.9532x10⁻⁶	4.005x10⁻¹¹	0.06440x10⁻⁶		0.8888x10⁻⁶		102.10
93.60	11.00	95.0	1408.00	164.57	8730.0	0.5044x10⁻⁶	1.609x10⁻¹¹	0.02587x10⁻⁶		0.4785x10⁻⁶		104.60

TABLE 2

Q cfs	$\frac{Q^2}{2g}$ = tan θ	Water-Surface Elevation at Sta 0+00	$\frac{v_o^2}{2g}$	Water-Surface Elevation in the Reservoir
600	0.0560x10⁵	102.330	0.005000	102.335
800	0.0995x10⁵	102.710	0.008012	102.718
1000	0.156x10⁵	103.055	0.01145	103.066
1200	0.224x10⁵	103.403	0.01505	103.418
1500	0.350x10⁵	103.810	0.02126	103.831
1800	0.504x10⁵	104.230	0.02769	104.258

FIGURE 5

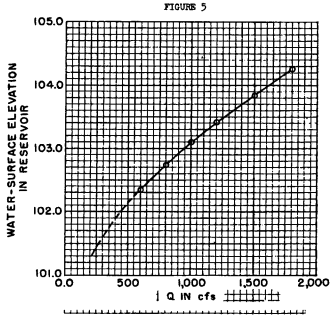
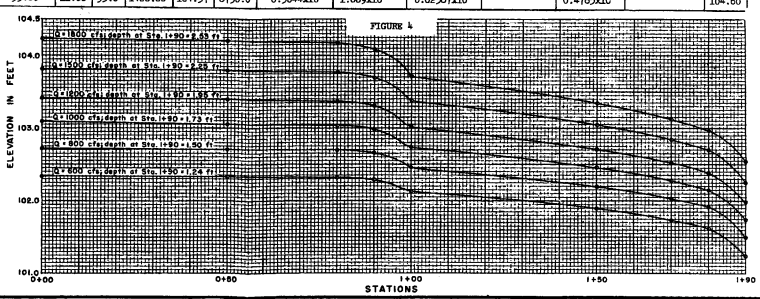


FIGURE 4



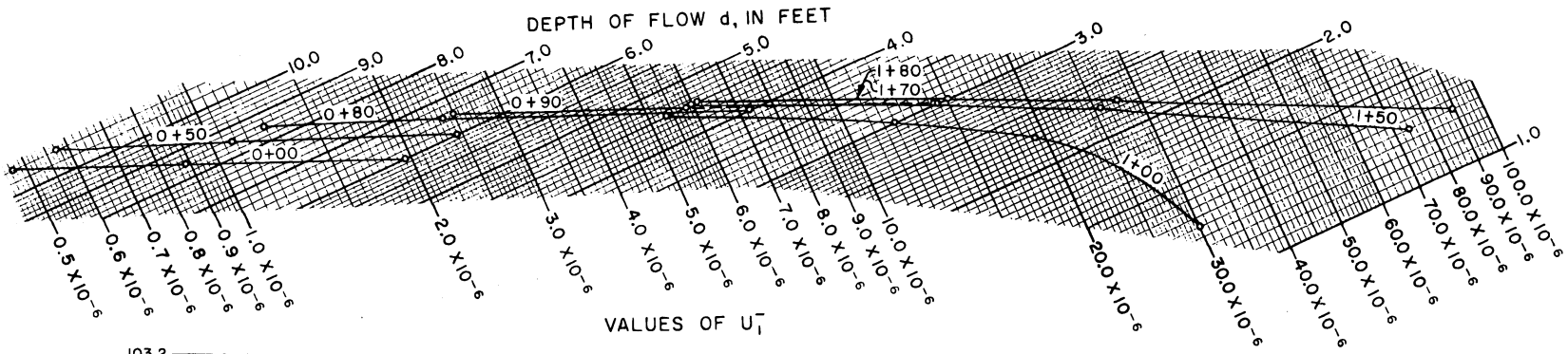
REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION- DESIGN SECTION

STANDARD DWG. NO.
ES – 83
SHEET 8 OF 10
DATE 8-6-54

HYDRAULICS: NON-UNIFORM FLOW IN A NON-PRISMATIC CHANNEL WITH LEVEL AND ADVERSE BOTTOM SLOPES
Example 6.

FIGURE 1



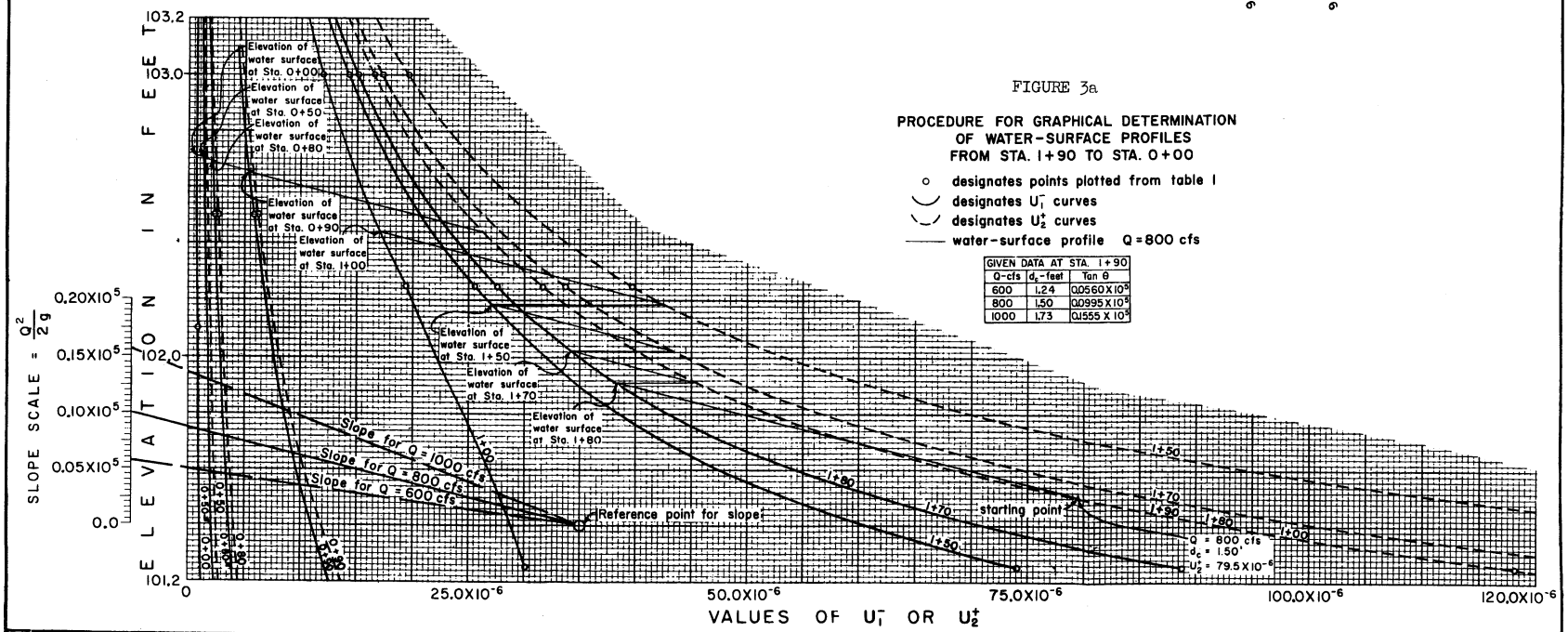


FIGURE 3a

PROCEDURE FOR GRAPHICAL DETERMINATION OF WATER-SURFACE PROFILES FROM STA. 1+90 TO STA. 0+00

- o designates points plotted from table I
- designates U_1^- curves
- designates U_2^+ curves
- —— water-surface profile Q = 800 cfs

GIVEN DATA AT STA. 1+90		
Q-cfs	d_c-feet	Tan θ
600	1.24	0.0560×10^{-5}
800	1.50	0.0995×10^{-5}
1000	1.73	0.1555×10^{-5}

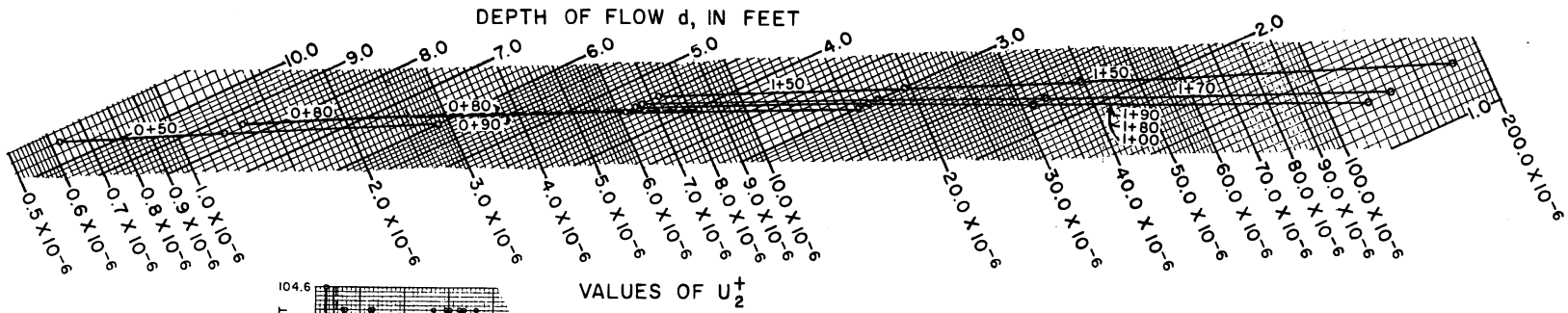
REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO. ES-83
SHEET 9 OF 10
DATE 8-6-54

HYDRAULICS: NON-UNIFORM FLOW IN A NON-PRISMATIC CHANNEL WITH LEVEL AND ADVERSE BOTTOM SLOPES —— Example 6.

FIGURE 2



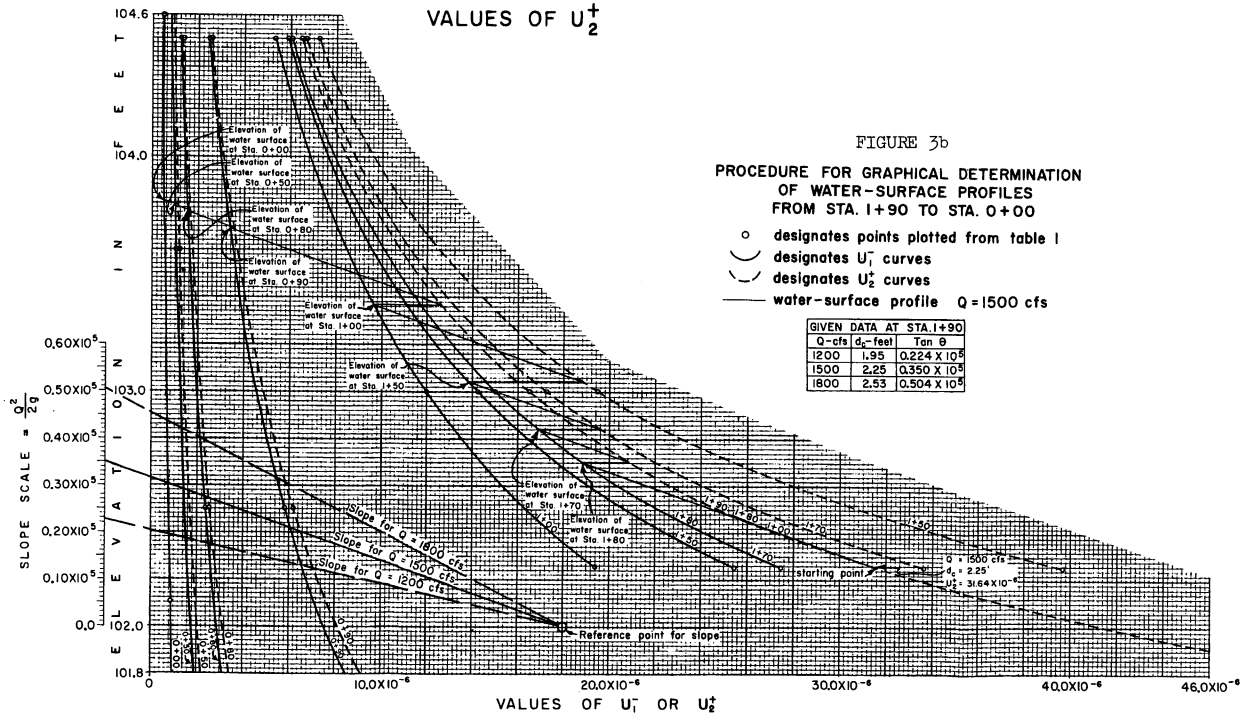


FIGURE 3b

PROCEDURE FOR GRAPHICAL DETERMINATION OF WATER-SURFACE PROFILES FROM STA. 1+90 TO STA. 0+00

o designates points plotted from table 1

designates U_1^- curves

designates U_2^+ curves

—— water-surface profile Q = 1500 cfs

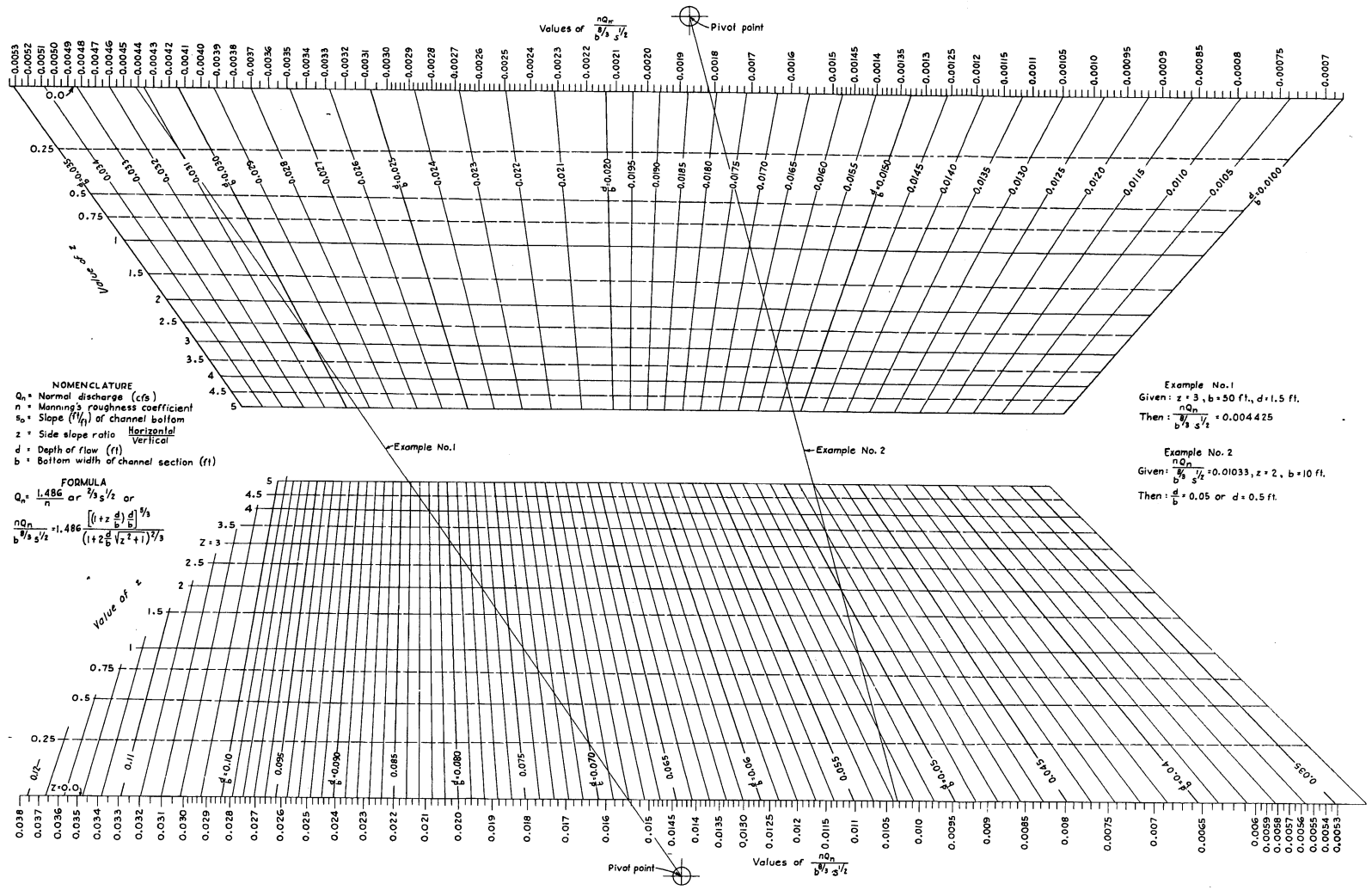
GIVEN DATA AT STA. 1+90		
Q-cfs	d_0-feet	Tan θ
1200	1.95	0.224 X 10^5
1500	2.25	0.350 X 10^5
1800	2.53	0.504 X 10^5

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO. ES-83
SHEET 10 OF 10
DATE 8-9-54

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
s_o = Slope (ft/ft) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2} \text{ or}$$

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1+z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1+2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 1

Given: $z = 3$, $b = 50$ ft., $d = 1.5$ ft.

Then: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 0.004425$

Example No. 2

Given: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 0.01033$, $z = 2$, $b = 10$ ft.

Then: $\frac{d}{b} = 0.05$ or $d = 0.5$ ft.

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

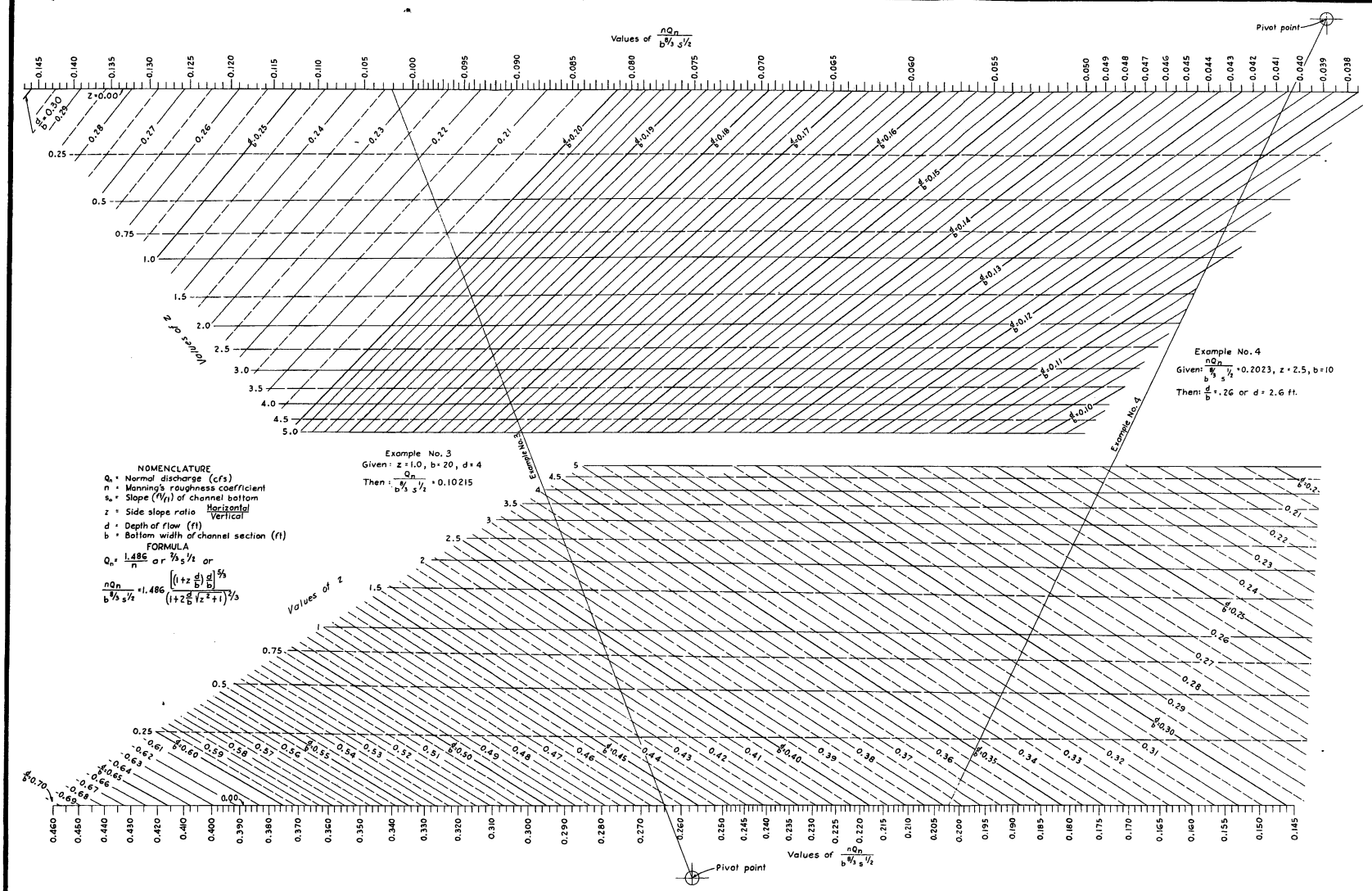
Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55
SHEET 1 OF 4
DATE 4-30-51

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
s_o = Slope (ft/ft) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2} \text{ or}$$

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1 + z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1 + 2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

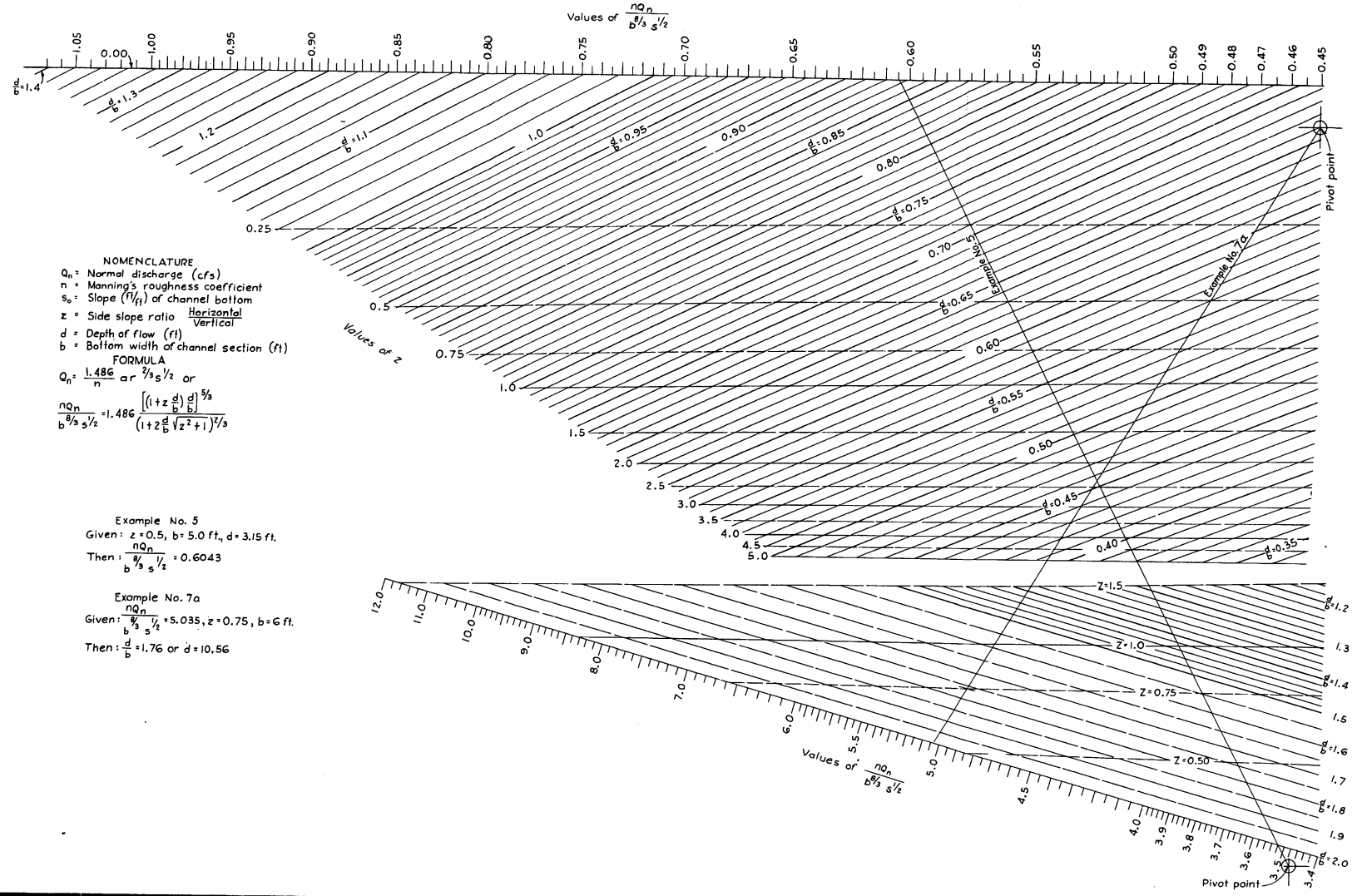
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55

SHEET 2 OF 4

DATE 4-30-51

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
s_o = Slope ($^{ft}/_{ft}$) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2} \text{ or}$$

$$\frac{nQ_n}{b^{8/3}s^{1/2}} = 1.486 \frac{\left[\left(1+z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1+2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 5

Given: $z = 0.5$, $b = 5.0$ ft., $d = 3.15$ ft.

Then: $\frac{nQ_n}{b^{8/3}s^{1/2}} = 0.6043$

Example No. 7a

Given: $\frac{nQ_n}{b^{8/3}s^{1/2}} = 5.035$, $z = 0.75$, $b = 6$ ft.

Then: $\frac{d}{b} = 1.76$ or $d = 10.56$

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

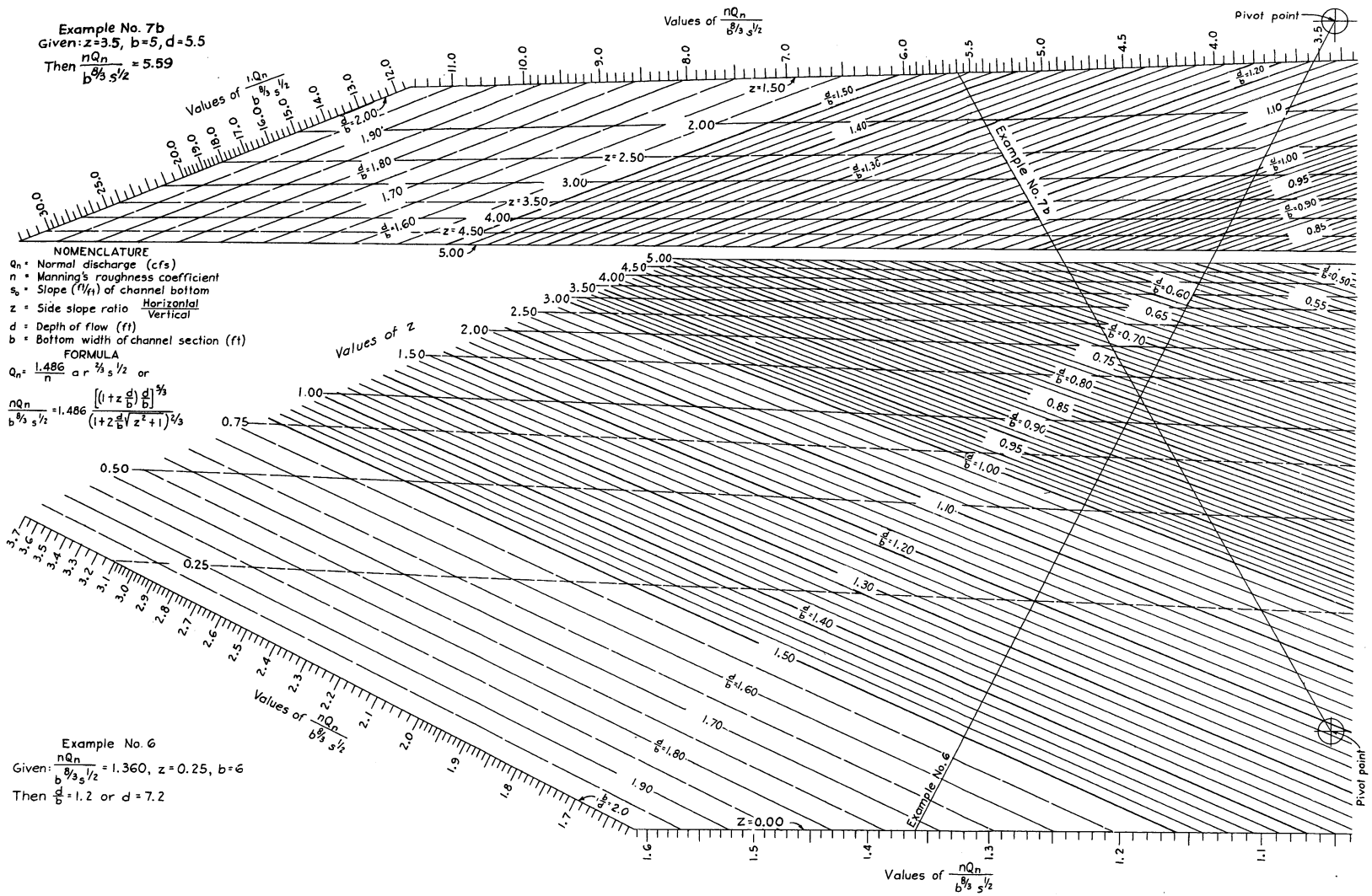
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55

SHEET 3 OF 4

DATE 4-30-51

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
s_o = Slope (ft/ft) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2} \quad \text{or}$$

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1+z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1+z\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

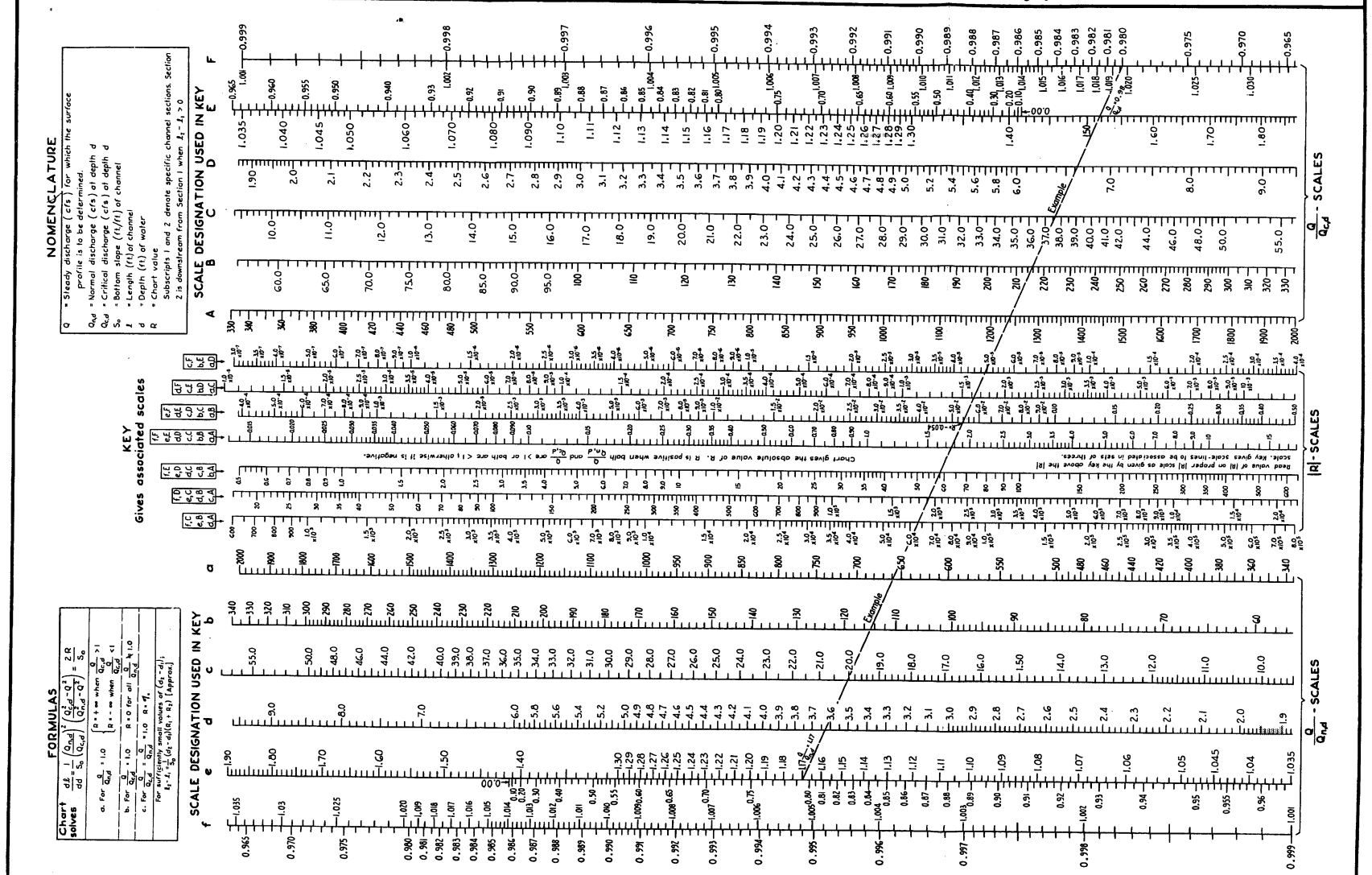
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55

SHEET 4 OF 4

DATE 4-30-51

HYDRAULICS: CHART FOR DETERMINING WATER SURFACE PROFILES FOR POSITIVE VALUE OF S_o ; STEP METHOD



REFERENCE

This nomogram was developed by Paul D. Doubt of the Engineering Standards Unit.

Revised 6-12-52

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-53

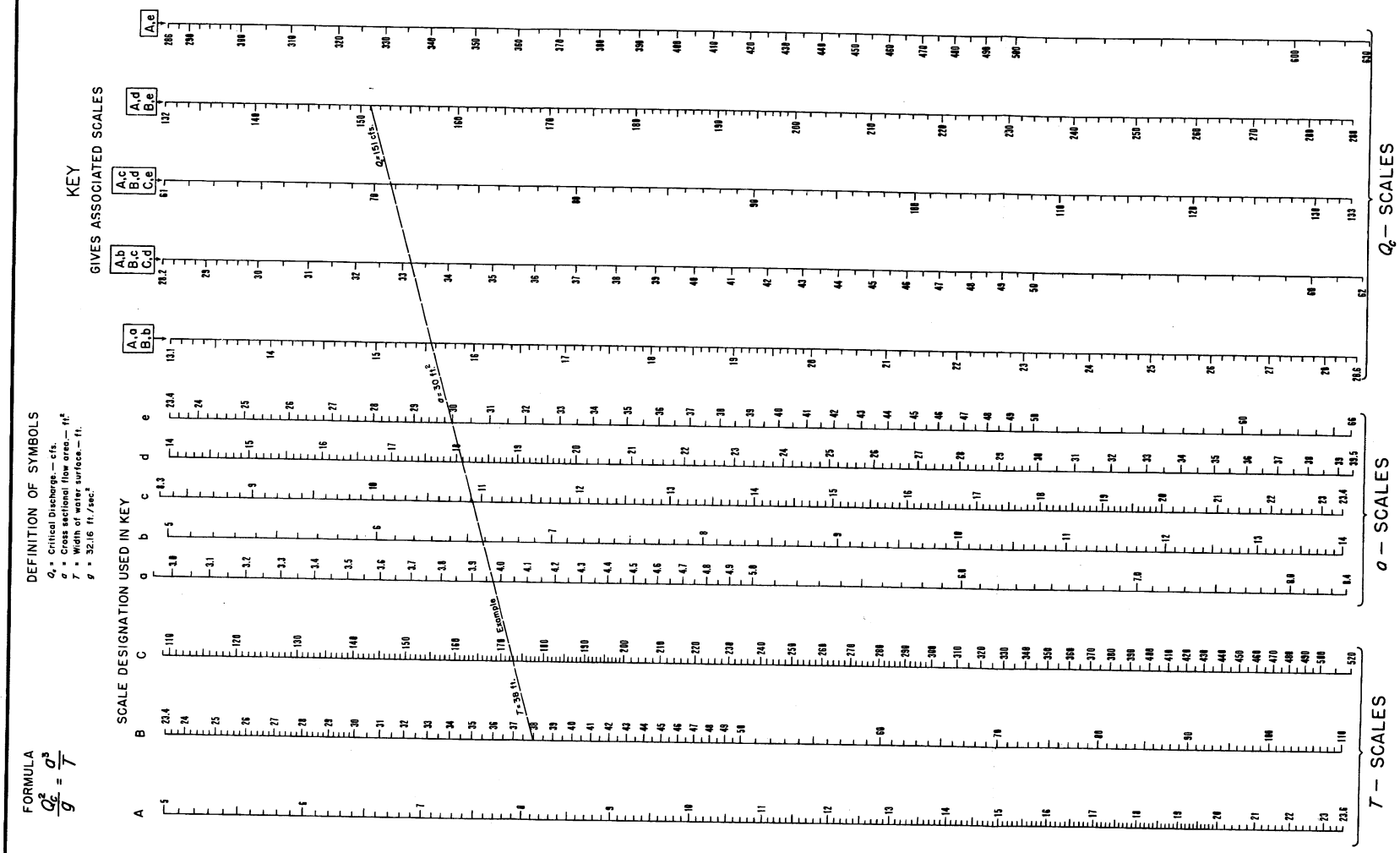
SHEET 1 OF 1

DATE 1-5-51

HYDRAULICS: SOLUTION OF GENERAL EQUATION FOR CRITICAL FLOW

T = 5 to 520
a = 3 to 66
Q_c = 13 to 630

Note that area values are repeated to insure solution on chart within stated range.



REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

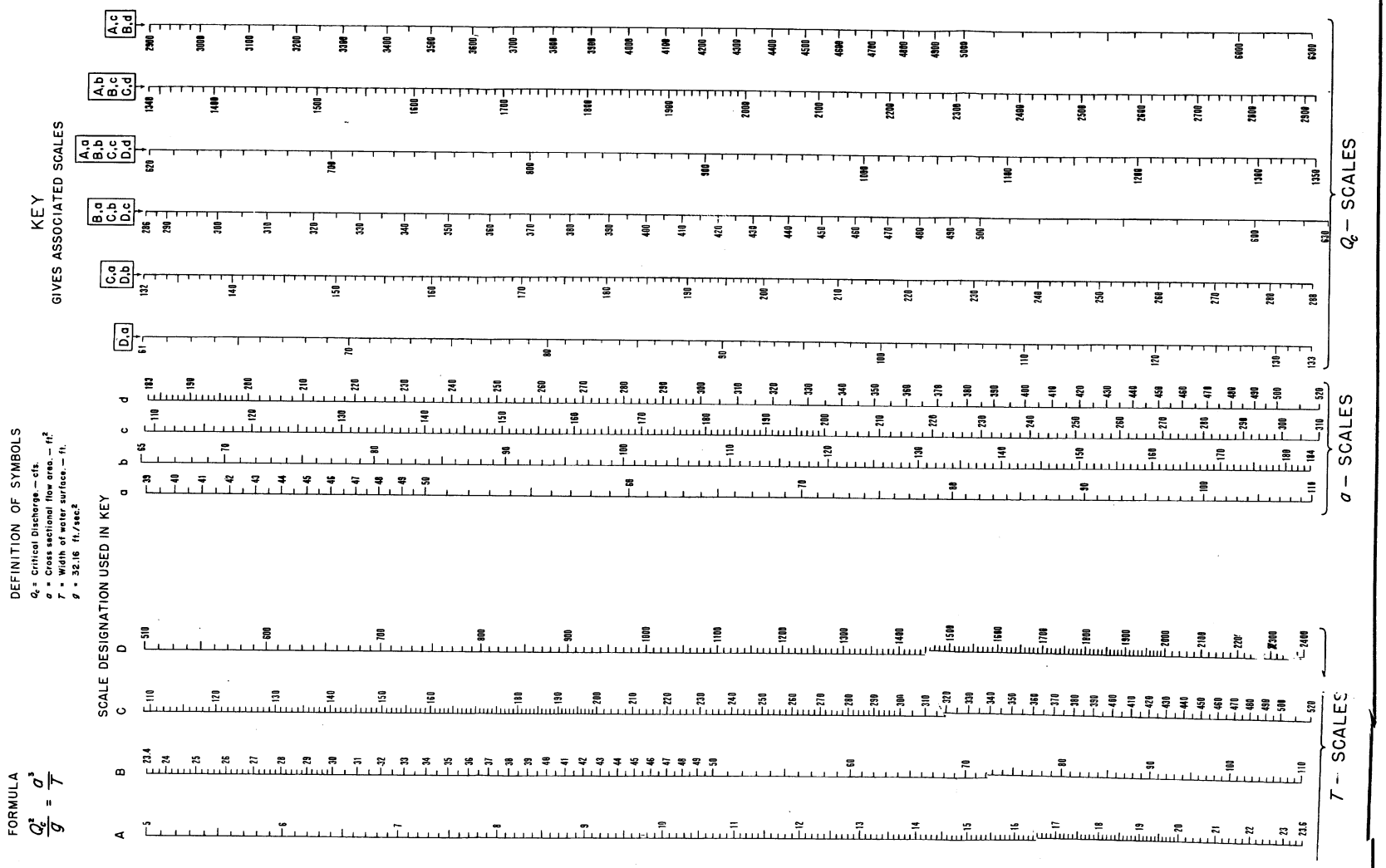
U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-75
SHEET 1 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF GENERAL EQUATION FOR CRITICAL FLOW

T = 5 to 2400
a = 39 to 520
Q_c = 61 to 6300

Note that area values are repeated to insure solution on chart within stated ranges.



REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

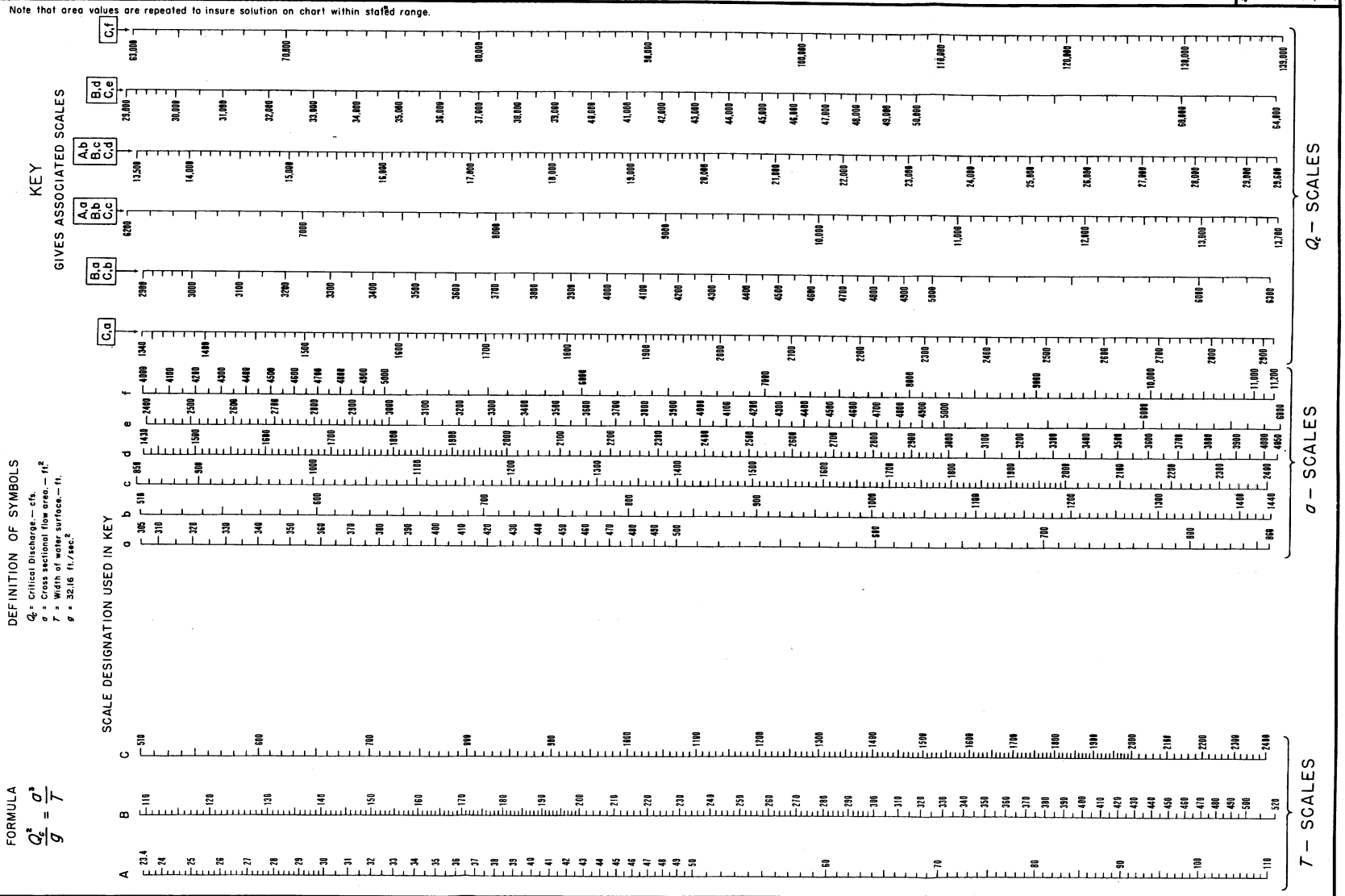
U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-75
SHEET 2 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF GENERAL EQUATION FOR CRITICAL FLOW

T = 23.4 to 2400
σ = 305 to 11,200
Q_c = 1340 to 139,000

Note that area values are repeated to insure solution on chart within stated range.



DEFINITION OF SYMBOLS

Q_c = Critical Discharge.— cfs.
σ = Cross sectional flow area.— ft.^{2}
T = Width of water surface.— ft.
g = 32.16 ft./sec.^{2}

FORMULA

$$\frac{Q_c^2}{g} = \frac{\sigma^3}{T}$$

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

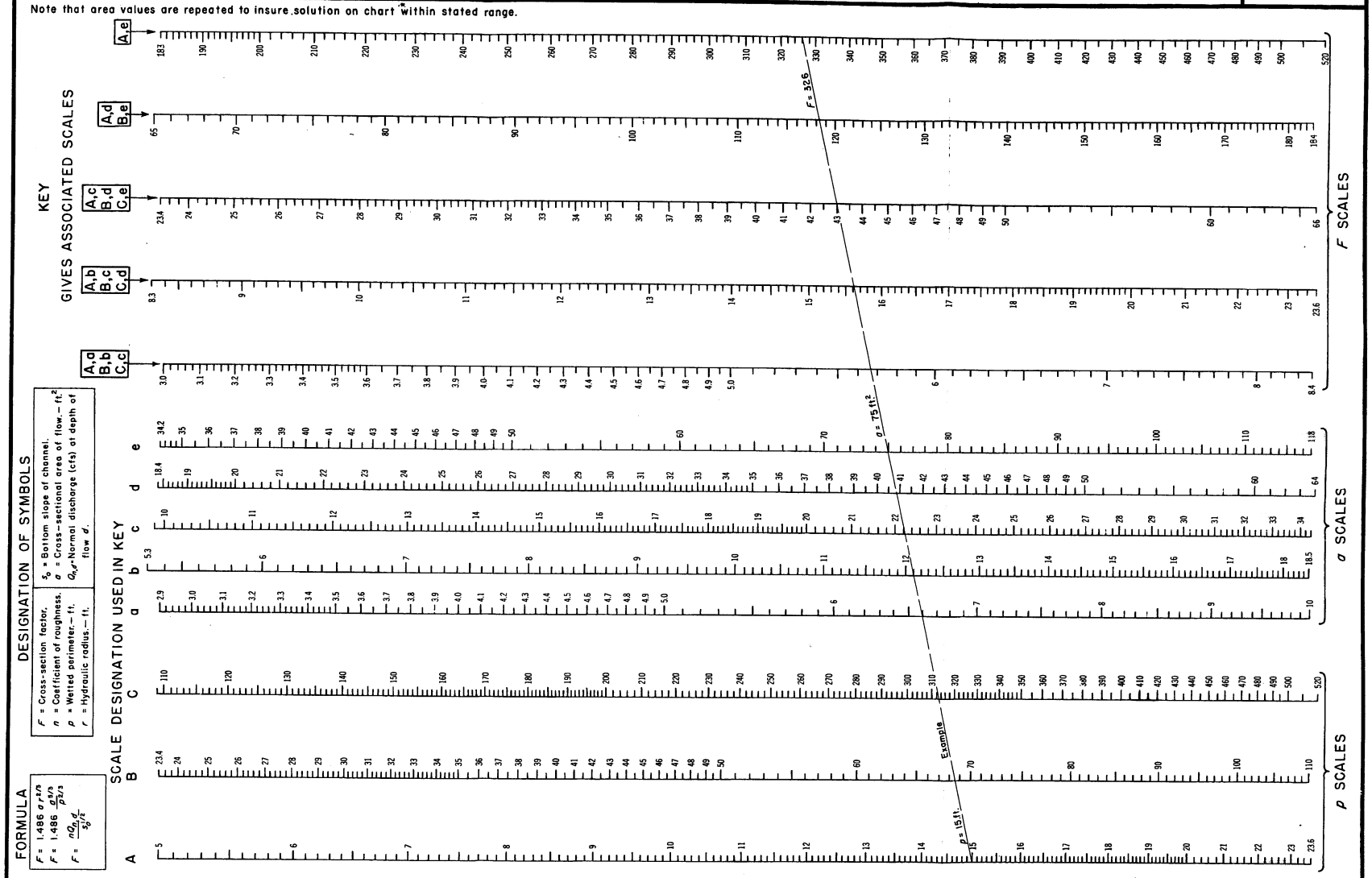
U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO ES-75
SHEET 3 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF CROSS SECTION FACTOR $F = \frac{nQ_{n,d}}{s_o^{1/2}} = 1.486\, a r^{2/3}$

p = 5 to 520
a = 2.9 to 118
F = 3 to 520

Note that area values are repeated to insure solution on chart within stated range.



DESIGNATION OF SYMBOLS

F = Cross-section factor.
n = Coefficient of roughness.
p = Wetted perimeter.— ft.
r = Hydraulic radius.— ft.

s_o = Bottom slope of channel.
a = Cross-sectional area of flow.— ft.²
$Q_{n,d}$ = Normal discharge (cfs) at depth of flow d.

FORMULA

$F = 1.486\, a r^{2/3}$
$F = 1.486\, \frac{a^{5/3}}{p^{2/3}}$
$F = \frac{nQ_{n,d}}{s_o^{1/2}}$

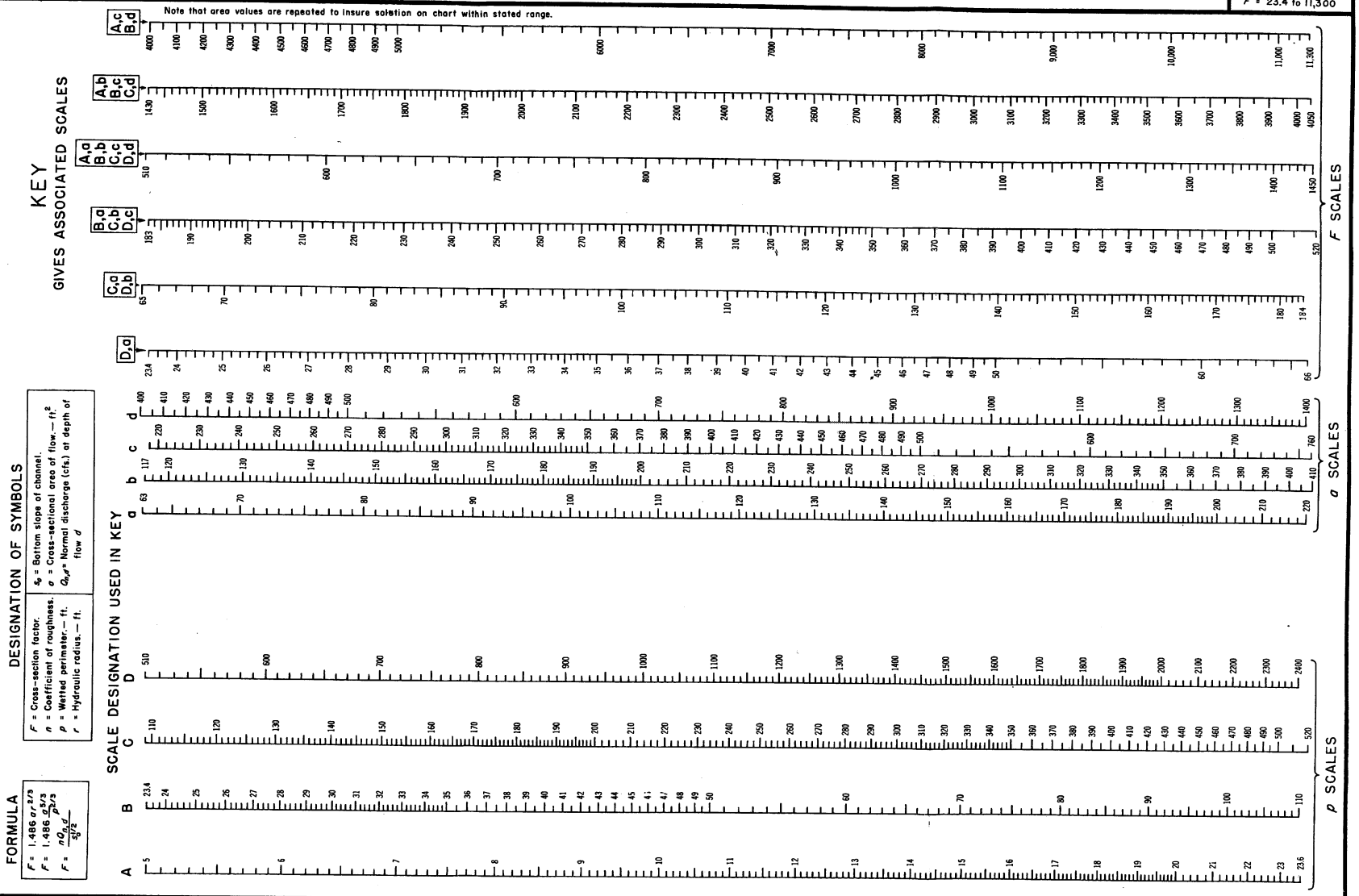
REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES 76
SHEET 1 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF CROSS SECTION FACTOR $F = \frac{nQ_{n,d}}{S_o^{1/2}} = 1.486\, a r^{2/3}$

p = 5 to 2400
a = 63 to 1400
F = 23.4 to 11,300



DESIGNATION OF SYMBOLS

F = Cross-section factor.
n = Coefficient of roughness.
p = Wetted perimeter.—ft.
r = Hydraulic radius.—ft.
S_o = Bottom slope of channel.
a = Cross-sectional area of flow.—ft^2
$Q_{n,d}$ = Normal discharge (cfs.) at depth of flow d

FORMULA

$F = 1.486\, a r^{2/3}$
$F = 1.486\, \frac{a^{5/3}}{p^{2/3}}$
$F = \frac{nQ_{n,d}}{S_o^{1/2}}$

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

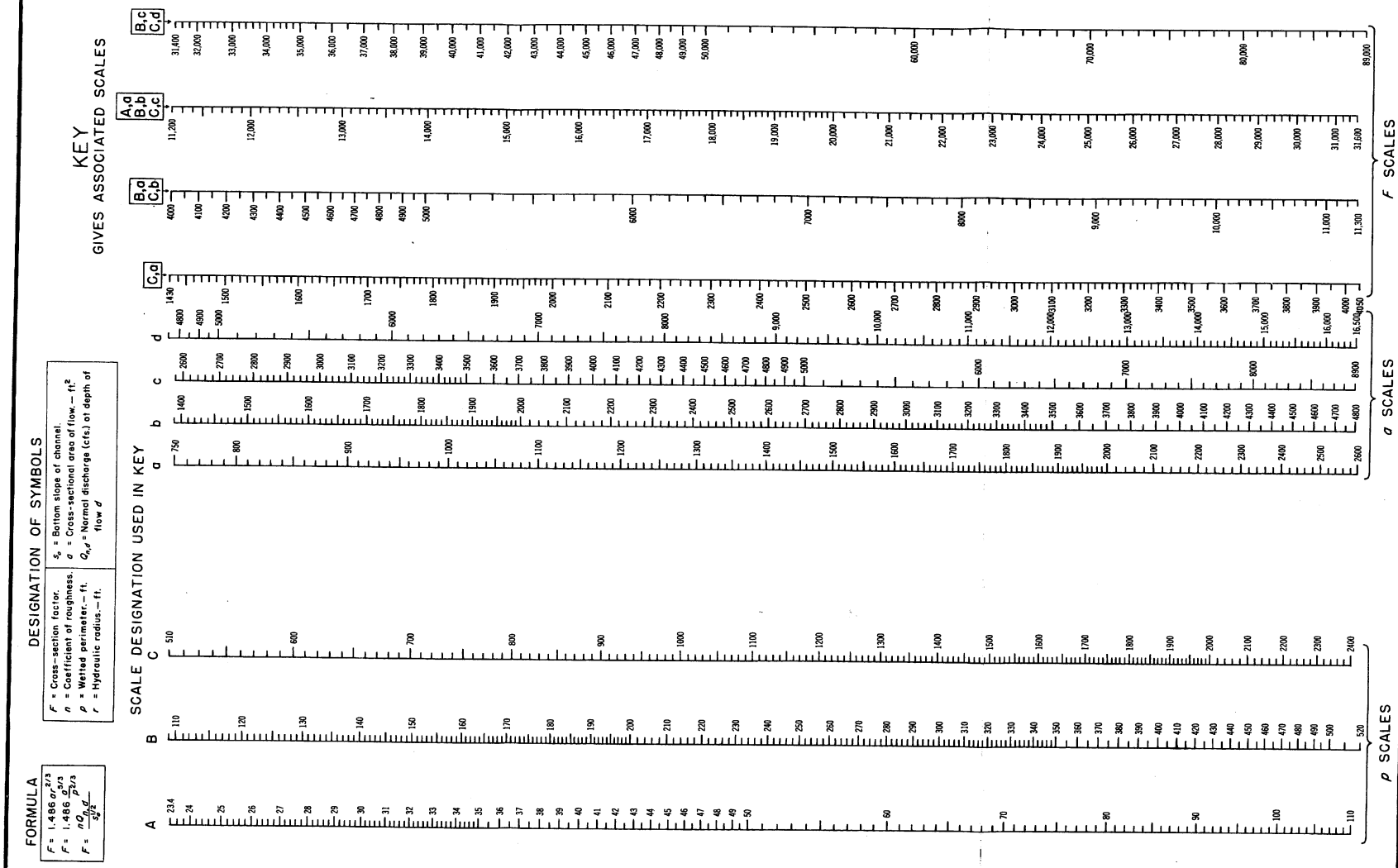
SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES 76
SHEET 2 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF CROSS SECTION FACTOR $F = \frac{nQ_{nd}}{S_o^{1/2}} = 1.486\, a r^{2/3}$

p = 23.4 to 2400
a = 750 to 16,500
F = 1430 to 89,000

Note that area values are repeated to insure solution on chart within stated range.



DESIGNATION OF SYMBOLS

F = Cross-section factor.	S_o = Bottom slope of channel.
n = Coefficient of roughness.	a = Cross-sectional area of flow.—ft.^{2}
p = Wetted perimeter.—ft.	Q_{nd} = Normal discharge (cfs.) at depth of flow d
r = Hydraulic radius.—ft.	

FORMULA

$F = 1.486\, a r^{2/3}$

$F = 1.486\, \frac{a^{5/3}}{p^{2/3}}$

$F = \frac{nQ_{nd}}{S_o^{1/2}}$

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION · DESIGN SECTION

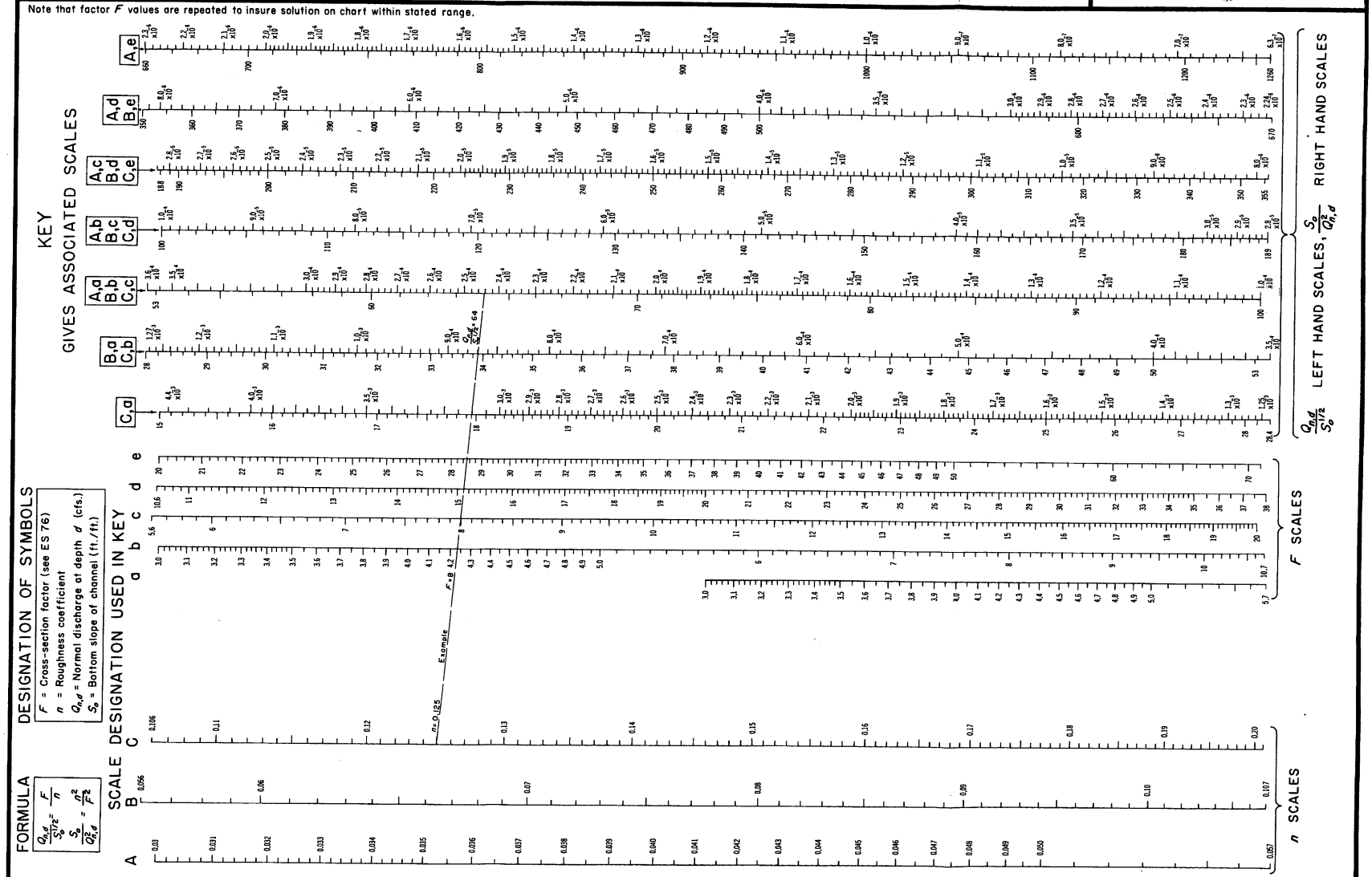
STANDARD DWG. NO. ES 76
SHEET 3 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} \text{ or } r^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 F = 3.0 to 70

$\frac{Q_{n,d}}{S_o^{1/2}}$ = 15 to 1260 $\frac{S_o}{Q_{n,d}^2}$ = 6.3×10^{-7} to 4.4×10^{-3}

Note that factor F values are repeated to insure solution on chart within stated range.



REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

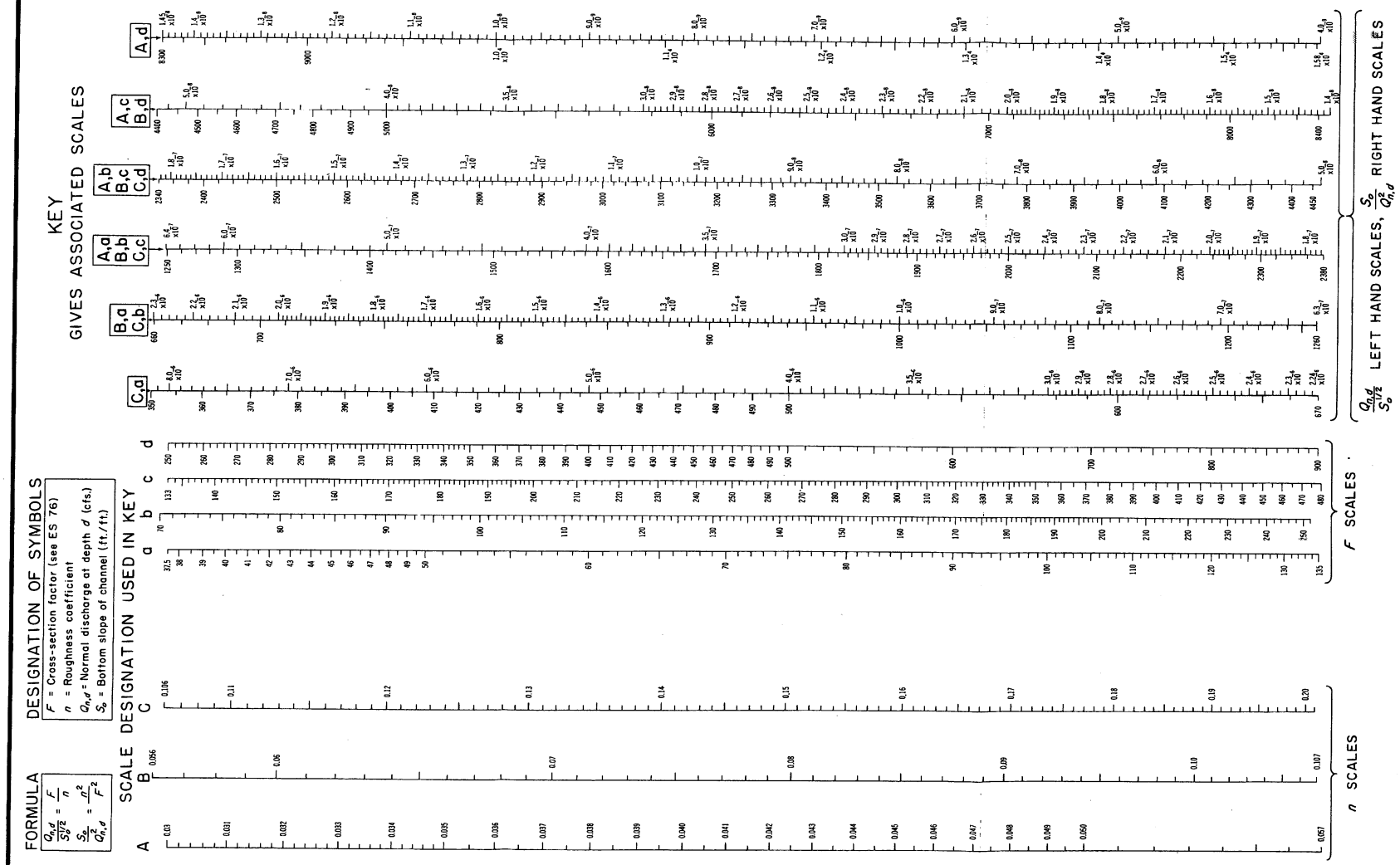
STANDARD DWG. NO. ES-77
SHEET 1 OF 4
DATE JULY 1953

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} a r^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 $\quad F$ = 38 to 900

$\frac{Q_{n,d}}{S_o^{1/2}}$ = 350 to 1.58×10^4 $\quad \frac{S_o}{Q_{n,d}^2}$ = 4.0×10^{-3} to 8.0×10^{-8}

Note that factor F values are repeated to insure solution on chart within stated range.



REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

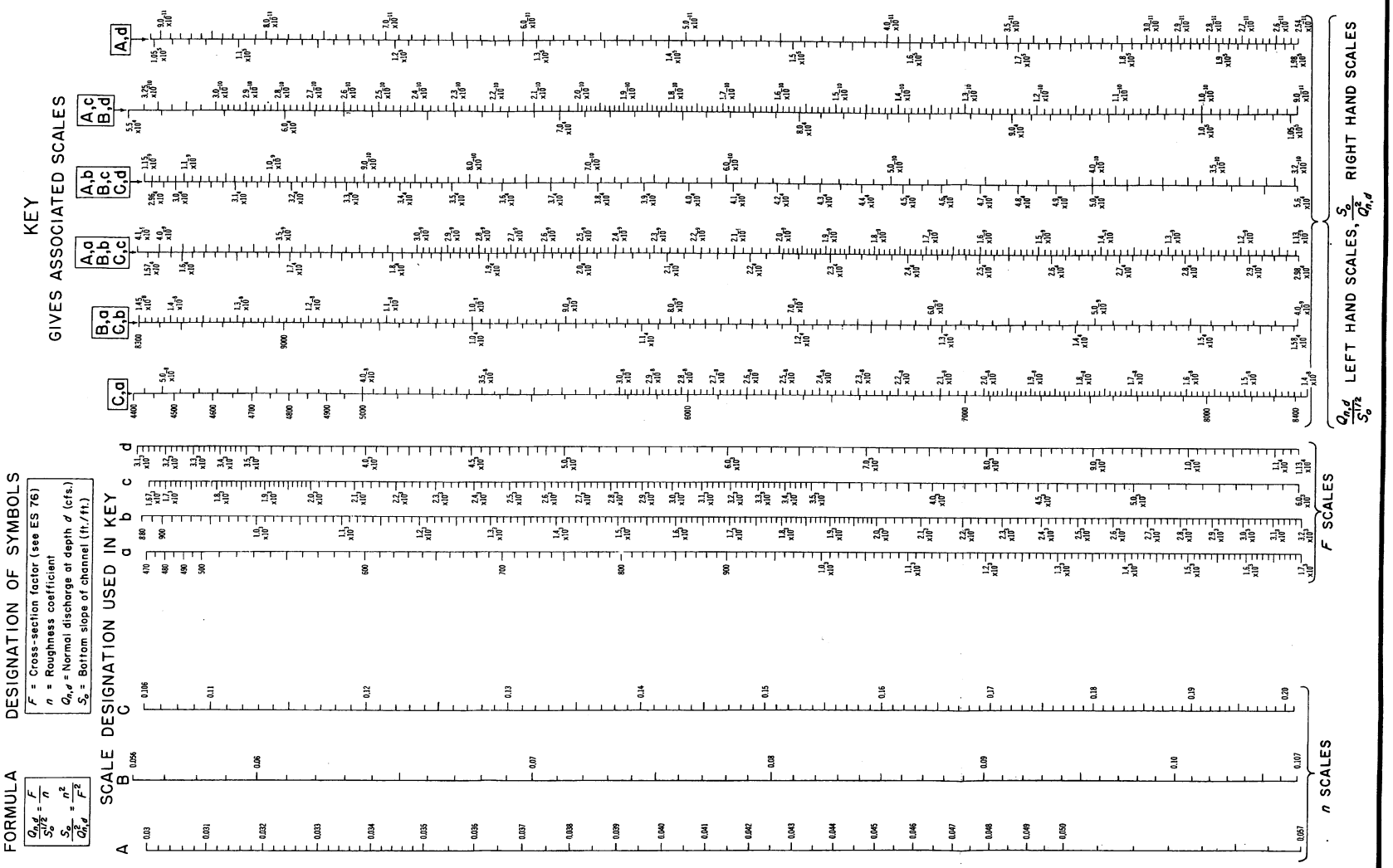
STANDARD DWG. NO. ES-77
SHEET 2 OF 4
DATE JULY 1953

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} a r^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 F = 470 to 1.1x10^4

$\frac{Q_{n,d}}{S_o^{1/2}}$ = 4400 to 1.98x10^5 $\frac{S_o}{Q_{n,d}^2}$ = 2.54x10^{-11} to 5.0x10^{-8}

Note that factor F values are repeated to insure solution on chart within stated range.



REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

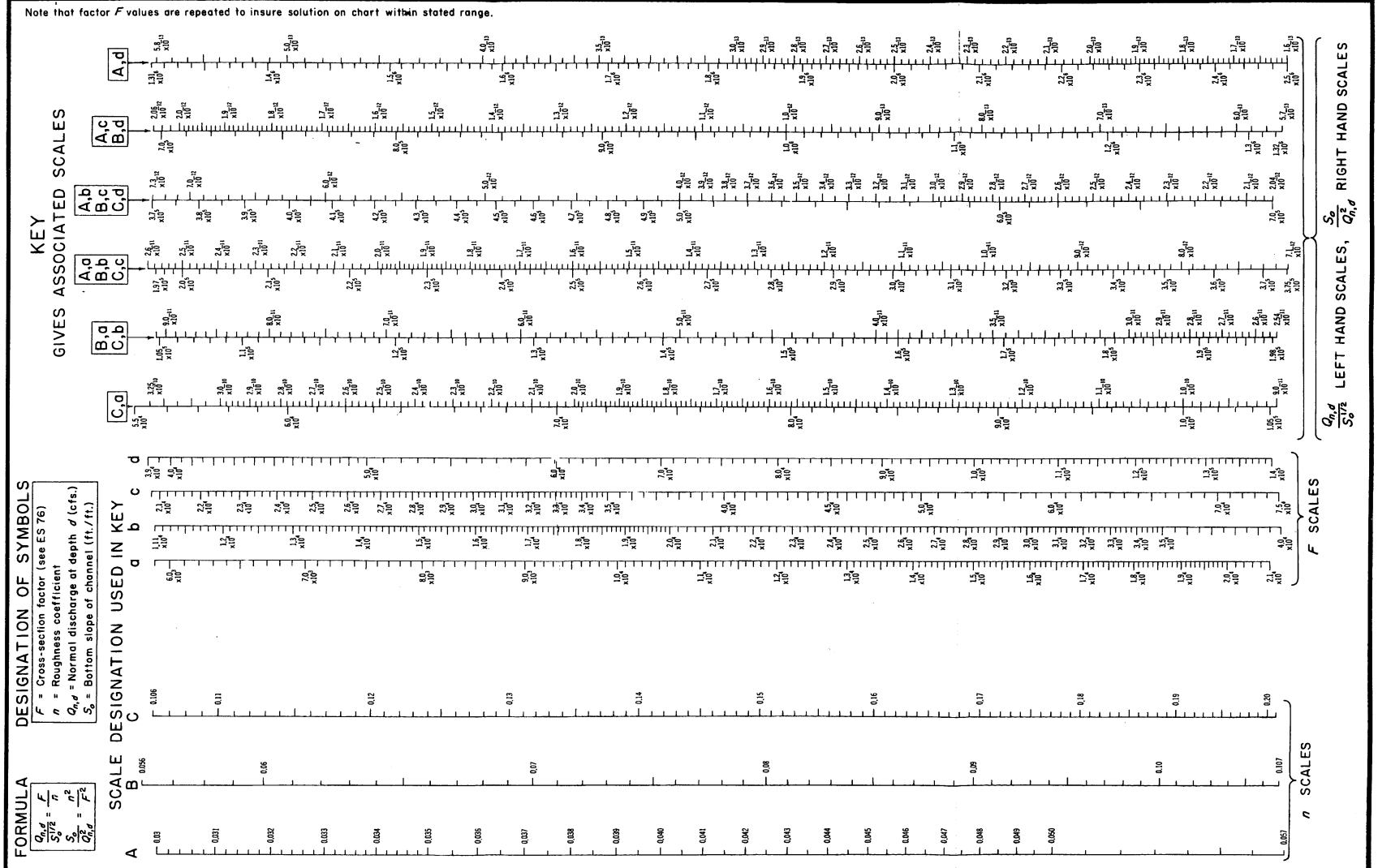
STANDARD DWG. NO. ES-77
SHEET 3 OF 4
DATE JULY 1953

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} a r^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 $F = 6.0 \times 10^3$ to 1.4×10^5

$\frac{Q_{n,d}}{S_o^{1/2}} = 5.5 \times 10^4$ to 2.5×10^6 $\frac{S_o}{Q_{n,d}^2} = 1.6 \times 10^{-13}$ to 3.25×10^{-10}

Note that factor F values are repeated to insure solution on chart within stated range.



FORMULA

$$\frac{Q_{n,d}}{S_o^{1/2}} = \frac{F}{n} \qquad \frac{S_o}{Q_{n,d}^2} = \frac{n^2}{F^2}$$

DESIGNATION OF SYMBOLS

F = Cross-section factor (see ES 76)

n = Roughness coefficient

$Q_{n,d}$ = Normal discharge at depth d (cfs.)

S_o = Bottom slope of channel (ft./ft.)

REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-77
SHEET 4 OF 4
DATE JULY 1953